Chapter 1

INTRODUCTION

1.1 Introduction

What is probability? [1]

CHANCE: general term of probability [2]

The probability problem often arises from a chance experiment:

An experiment(trial [3]) $\longrightarrow$ outcome

Example 1.1

Flipping coins and casting dice:

Figure 1.1: The game of dice casting.

[1] We start this course with probabilty, and the conceptw will evolve to random variables, and finally to the random processes near the end of the semester.

[2] In our daily life, we rather use the term "chance", while the term "probability" is often being used for academic purposes.

[3] A single performane of cahnce experiment is called "trial".

Question:

What is the chance of getting 2 or 3 showing up when you cast a die?

Solution: The answer is $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$!!! (but, how come and why???)

You get the answer very easily without a serious thought process like you memorize the multiplication table. In a simple problem of probability, such as this example, the way of getting the probability of an experiment is easy, but as the problem becomes more and more complex, we need a well structured way of solving the problem.

CONSTRUCTION OF A MATHEMATICAL MODEL:

(for an experiment): [4]

General problem $\Longrightarrow$ Math. structure

Three basic components:

1. **Sample sapce(S):**

   Set of all possible outcomes in any given experiment.

   (e.g.) In a game of casting a die, the sample sapce S consists of 6 outcomes as;

   $$S = \{a_1, a_2, a_3, a_4, a_5, a_6\}$$

   where a_i represents a single outcome.

2. **Event(A):**

   A specific (combination of) outcome(s) that we want to compute its probability. [5]

   (e.g.) In a game of casting a die, as a specific event that might occur during the game, we can consider the following case;

   $$A = \{a_2, a_4, a_6\} = \{\text{even numbers showing up}\}$$

[4] This is done by way of using the **set theory**.

[5] This corresponds to the subset of S in set theory.

3. **Probability(P):**

   A function of events assigning a non-negative number to each event.

   (e.g.) In a game of casting a die, we usually assign the probability as follows, if the die can be regarded as "fair";

$$P(a_i) = \frac{1}{6} \quad \forall\, i = 1, 2, \ldots, 6.$$