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Chapter 11

Practical Application : RADAR

Radar Detection using a single observation (active radar):

Figure 11.1: Active radar system environment.

(1) **Detection**[1] existence of $s(t)$ in $r(t)$

$$\begin{cases} H_0 & : & r(t) = n(t) \\ H_1 & : & r(t) = \alpha s(t) + n(t) \end{cases}$$

(2) **Estimation:** measuring L

Distance[2] from the radar to the target : $\widehat{L}$

Typical $s(t)$: train of pulses [3]

Figure 11.2: Active radar system environment.

[1] This type of problem is called the "hypothesis testing":

[2] Or, we could estimate the round-trip propagation delay of $s(t)$: $\widehat{D}$, and then the estimate of the distance is $\widehat{L} = \frac{\widehat{D}}{2} \cdot C$ where C is the signal velocity.

[3] T_p is called the PRI(pulse repetition interval), and $f_p \triangleq \frac{1}{T_p}$ is called the PRF(pulse repetition frequency).

11.1 Detection

Figure 11.3: The block diagram of detection in RADAR.

Two types of error:

(a) **False alarm:** decide H_1 when no target is present

$$P(\text{false alarm}) \triangleq P_{fa} = P\{\text{decide} H_1 \mid H_0 \text{is true}\}$$
$$= \int_{W_T}^{\infty} f_0(w)dw$$

(b) **Miss:** decide H_0 when the target is present

$$P(\text{miss}) \triangleq P_m = P\{\text{decide} H_0 \mid H_1 \text{is true}\}$$
$$= 1 - P(\text{detection})$$
$$= 1 - \int_{W_T}^{\infty} f_1(w)dw$$

where $f_0(\omega)$ and $f_1(\omega)$ are the conditional p.d.f. of $W(t)$ when the target is absent and present respectively, and W_T corresponds to the decision threshold.

Figure 11.4: The conditional p.d.f.'s of $W(t)$ under H_0 and H_1.

Design parameters: determination[4] of W_T

We want:

(1) P_{fa} : as small as possible

(2) P_m : as small as possible [5]

CFAR[6] alogorithm:

: Given a fixed desired value of P_{fa}, find the threshold W_T

(1) P_{fa}:

The p.d.f. of the envelop R of the received signal $r(t)$ when only the noise is present, is known to have the Rician distribution as follows:

$$f_{R_0}(r) = \frac{re^{-r^2/2\sigma^2}}{\sigma^2} \cdot u(r)$$

where σ^2 is the noise power.

Then, the probability of false alarm becomes:

$$\begin{aligned} P_{fa} &= \int_{W_T}^{\infty} f_0(w)dw = \int_{R_T}^{\infty} f_{R_0}(w)dw \\ &= \int_{R_T}^{\infty} \frac{re^{-r^2/2\sigma^2}}{\sigma^2}dr \\ &= e^{-R_T^2/2\sigma^2} \end{aligned}$$

where $R_T = g^{-1}(W_T)$, since $W_T = g(R_T)$ and $g(\cdot)$ is monotonic (i.e. $\exists$ one-to-one mapping).

Solving above equation w.r.t. R_T, we get:

$$R_T = \sqrt{2\sigma^2 \ln\left(\frac{1}{P_{fa}}\right)}$$

And the threshold W_T is determined as:

$$W_T = g\left(\sqrt{2\sigma^2 \ln\left(\frac{1}{P_{fa}}\right)}\right)$$

[4]The rest of the system is just a hardware!

[5]This is equivalent to make P_d as large as possible.

[6]CFAR : constant false alarm rate)

(2) P_d: w/ fixed W_T

The p.d.f. of the envelop R of the received signal $r(t)$ when the target and the noise are both present, is known to have the Rician distribution as follows:

$$f_{R_1}(r) = \frac{r}{\sigma^2} I_0\left(\frac{rA_0}{\sigma^2}\right) e^{-(r^2+A_0^2)/2\sigma^2}$$

where σ^2 and $\frac{A_0^2}{2}$ are the noise and signal powers respectively, and $I_0(\beta)$ is the Bessel function of zero order given as follows:

$$I_0(\beta) = \frac{1}{2\pi}\int_0^{2\pi} e^{\beta\cos(\theta)} d\theta$$

Then, the probability of detection becomes:

$$\begin{aligned} P_d = \int_{W_T}^{\infty} f_1(w)dw &= \int_{R_T}^{\infty} f_{R_1}(w)dw \\ &= \int_{\sqrt{2\sigma^2\ln(P_{fa}^{-1})}}^{\infty} f_{R_1}(w)dw \\ &= Q\left(\sqrt{\frac{A_0^2}{\sigma^2}}\, ,\, \sqrt{2\ln(P_{fa}^{-1})}\right) \end{aligned}$$

where $Q(\alpha,\beta)$ is the Marcum's Q-function defined as:

$$Q(\alpha,\beta) \triangleq \int_{\alpha}^{\beta} \gamma I_0(\alpha\gamma) e^{-(\gamma^2+\alpha^2)/2} d\gamma$$

Note: Refer to Fig. 10.7-2 of the textbook

(i) P_d is a function of SNR $= \frac{A_0^2}{2\sigma^2}$ w/ P_{fa} parameter.

(ii) The smaller P_{fa} we choose, the smaller P_d becomes, which is the price we have to pay.

(iii) Smaller P_{fa} with the same P_d provides larger SNR.

(iv) With small P_{fa} and large P_d, the detection probability can be approximated as:

$$P_d \approx F\left\{\frac{A_0}{\sigma} - \sqrt{2\ln(P_{fa}^{-1})}\right\}$$

where $F(\cdot)$ is the standard Gaussian distribution function.

11.2 Estimation

Figure 11.5: The block diagram of time delay estimation in RADAR.

FACT: $x(t+t_1) \odot y(t+t_2) = R_{XY}(\tau + t_2 - t_1)$

where $\odot$ represents the cross-correlation.

proof:

$$\begin{aligned}
\text{LHS} &= \frac{1}{T}\int_T x(t+t_1)y(t+t_2+\tau)dt \\
& \quad (\text{let } t+t_1 = t') \\
&= \frac{1}{T}\int_T x(t')y(t^{'}+\tau+t_2-t_1)dt^{'} \\
&= R_{XY}(\tau+t_2-t_1) \\
&= \text{RHS}
\end{aligned}$$

$$R_{sr}(\tau) = s(t) \odot \alpha s(t-D) = \alpha R_{ss}(\tau - D)$$

Figure 11.6: The siganl, auto, and cross-correlation functions.

Note:

(1) By detecting the location of the peak in $R_{sr}(\tau)$ within T_p, we can estimate the time delay $\widehat{D}$:

$$\widehat{D} = \underset{\tau}{argmax}\ R_{sr}(\tau)$$

and corresponding distance b/w the radar site and the target L follows as:

$$\widehat{L} = \frac{\widehat{D}}{2} \cdot C$$

(2) To avoid the ambiguity among adjacent pulses, the condition below should be satisfied:

$$D \leq T_p$$

Therefore, for a certain radar system, the maximum detection range L_{max} is determined by:

$$L_{max} = \frac{T_p}{2} \cdot C \propto T_p$$

Remark: peak detection $\longrightarrow$ zero-crossing detection [7]

Figure 11.7: Peak detection vs. zero-crossing detection for $\widehat{D}$.

[7] This provides less ambiguous process of finding D.