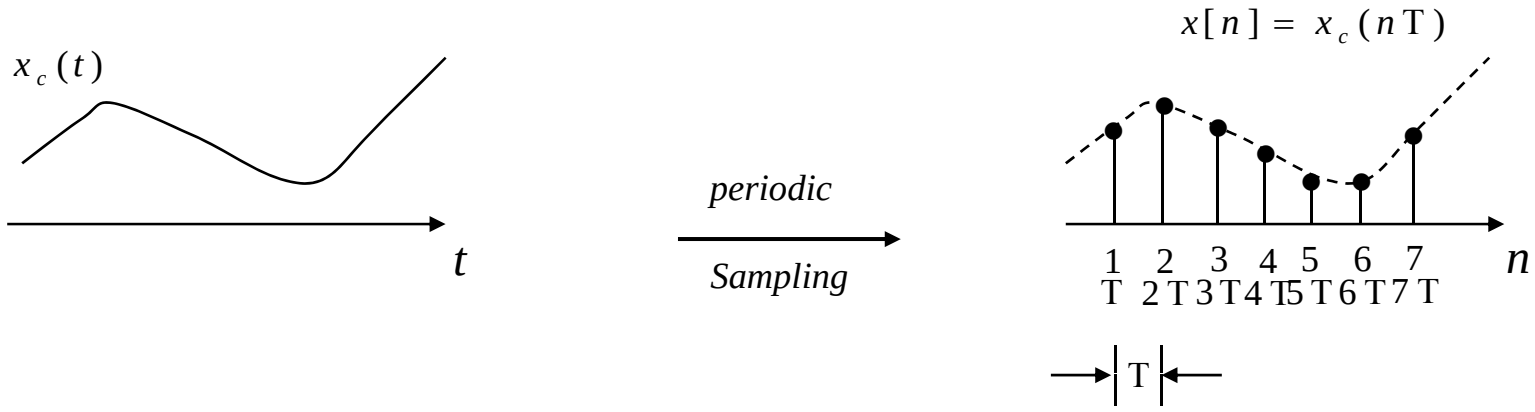


Figure 4.1

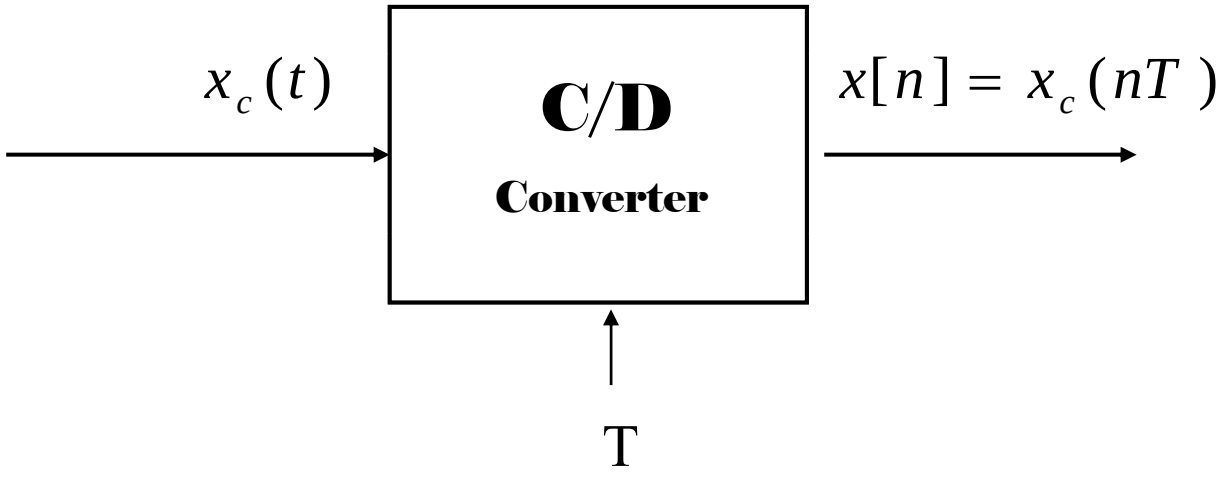
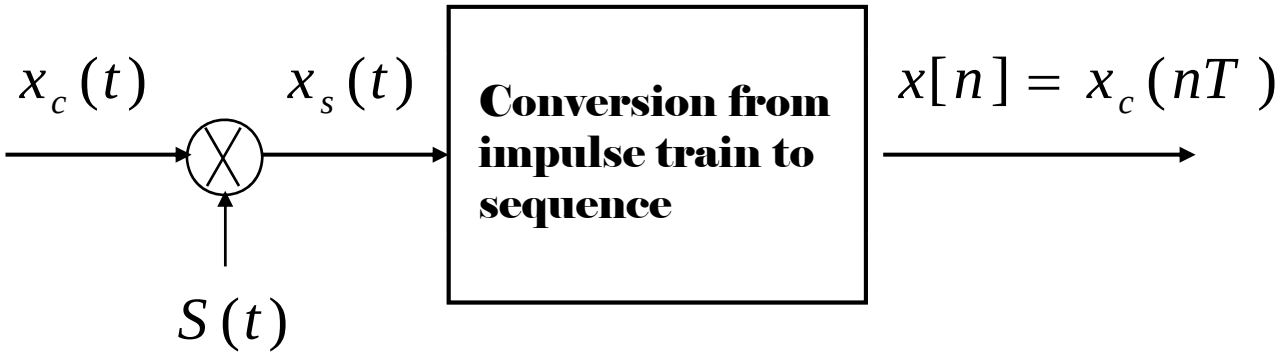


Figure 4.2



(i.e. area of $\delta(t)$ magnitude of $\delta[n]$)

Figure 4.3

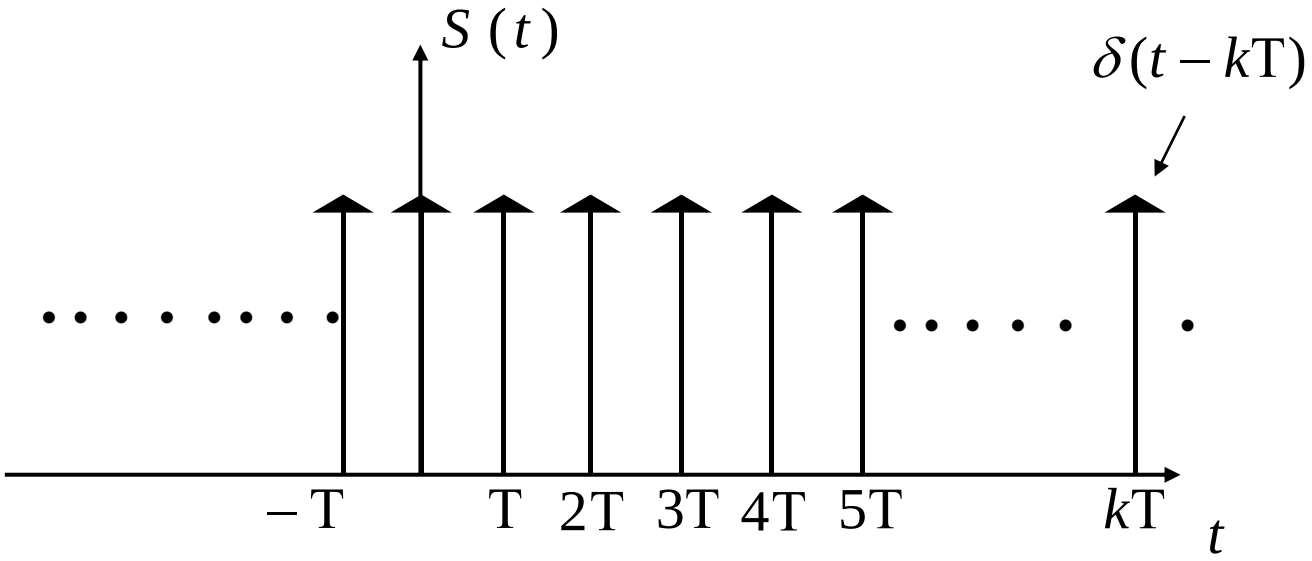


Figure 4.4

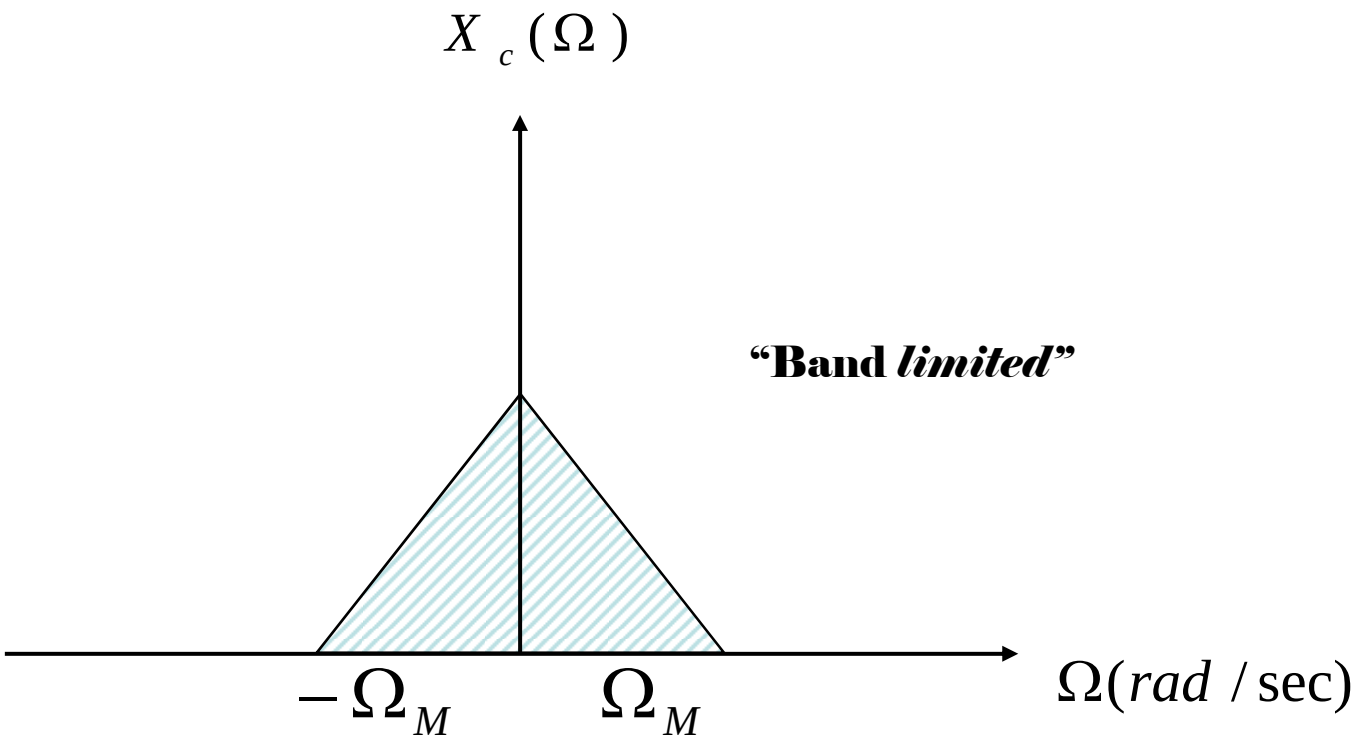


Figure 4.5

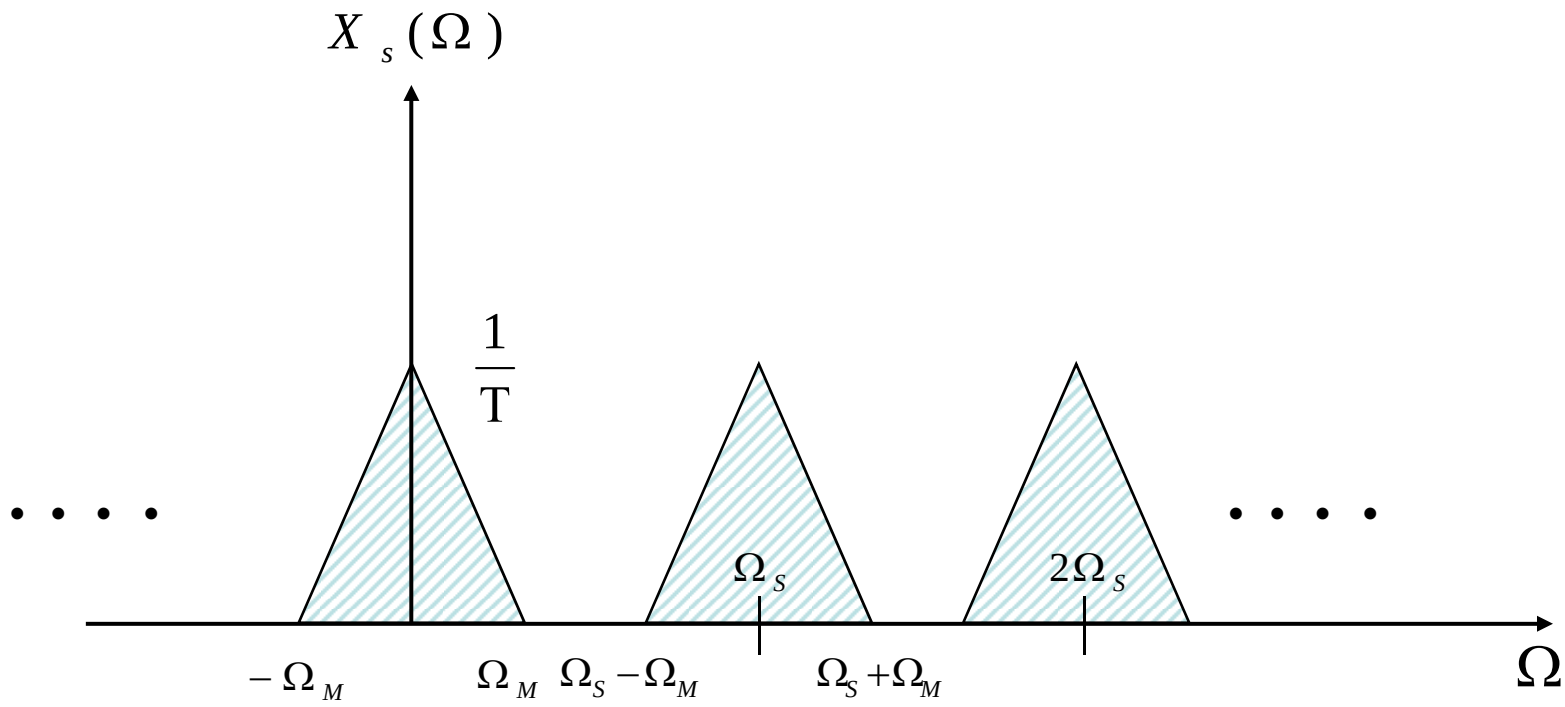

$X_s(\Omega)$
$\frac{1}{T}$
Ω_S
$2\Omega_S$
$-\Omega_M$
Ω_M
$\Omega_S-\Omega_M$
$\Omega_S+\Omega_M$
Ω

Figure 4.6

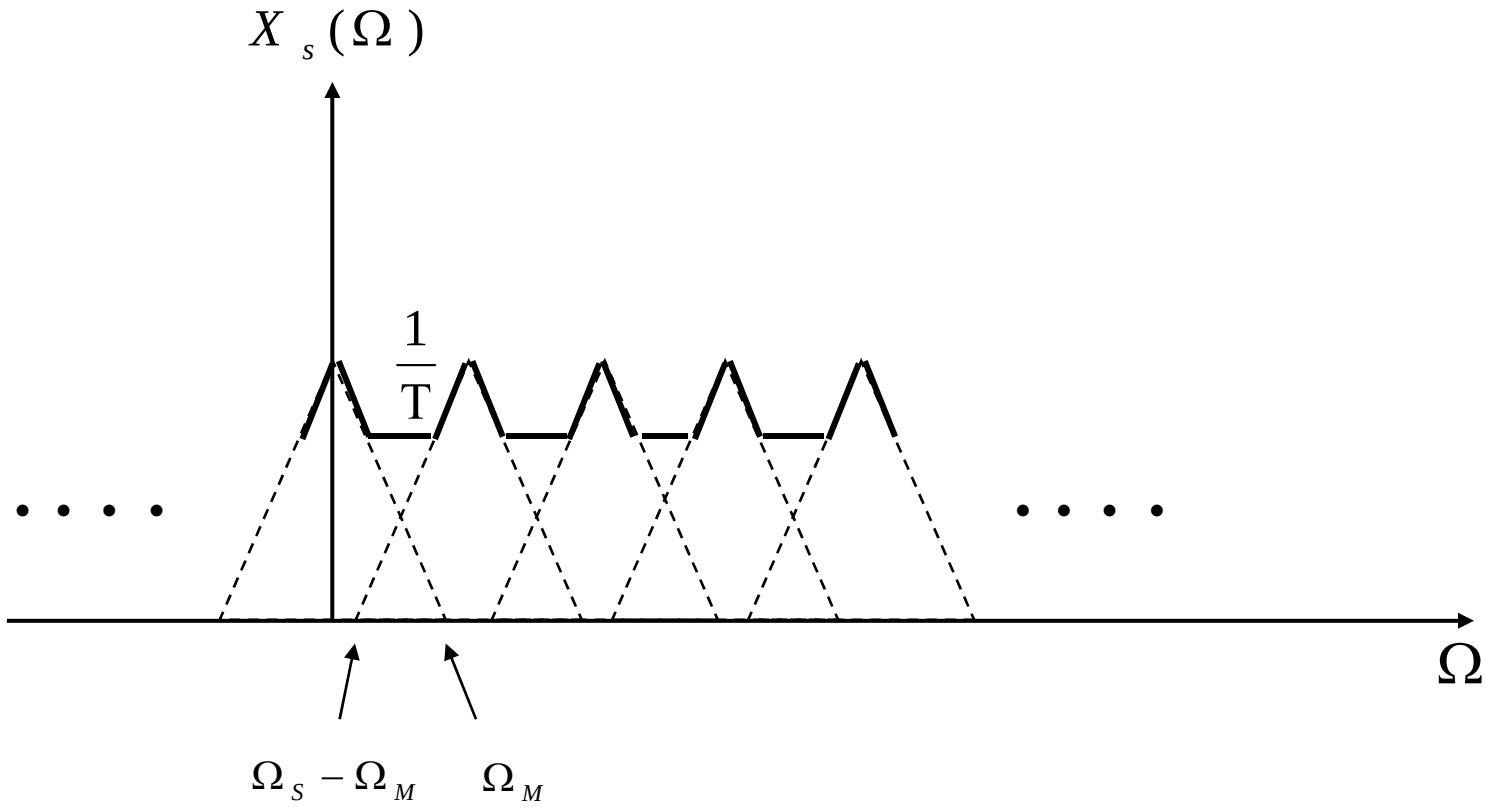


Figure 4.7

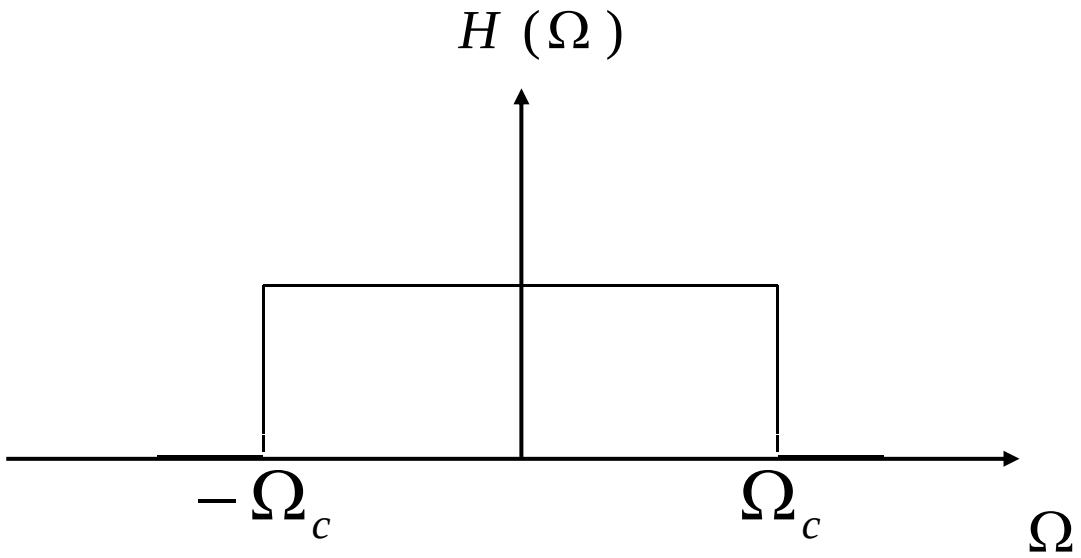


Figure 4.8

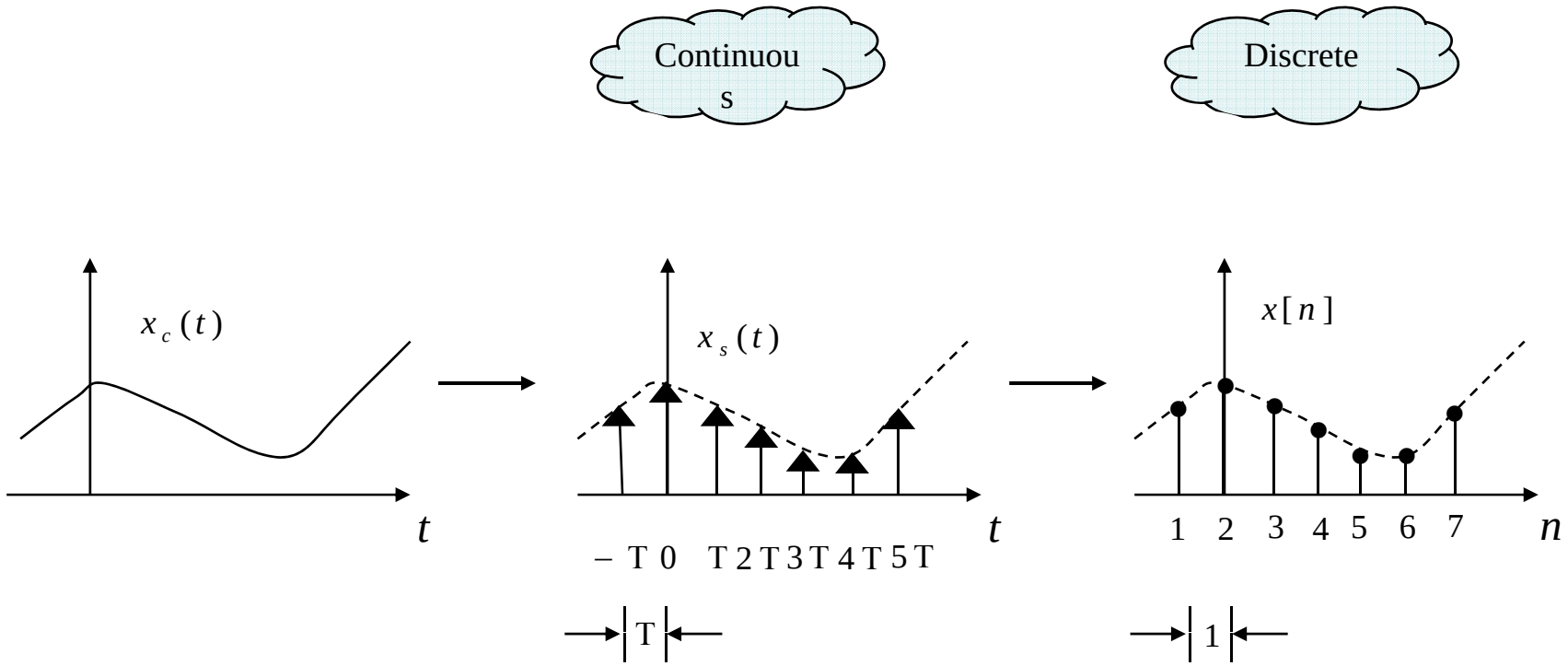


Figure 4.9

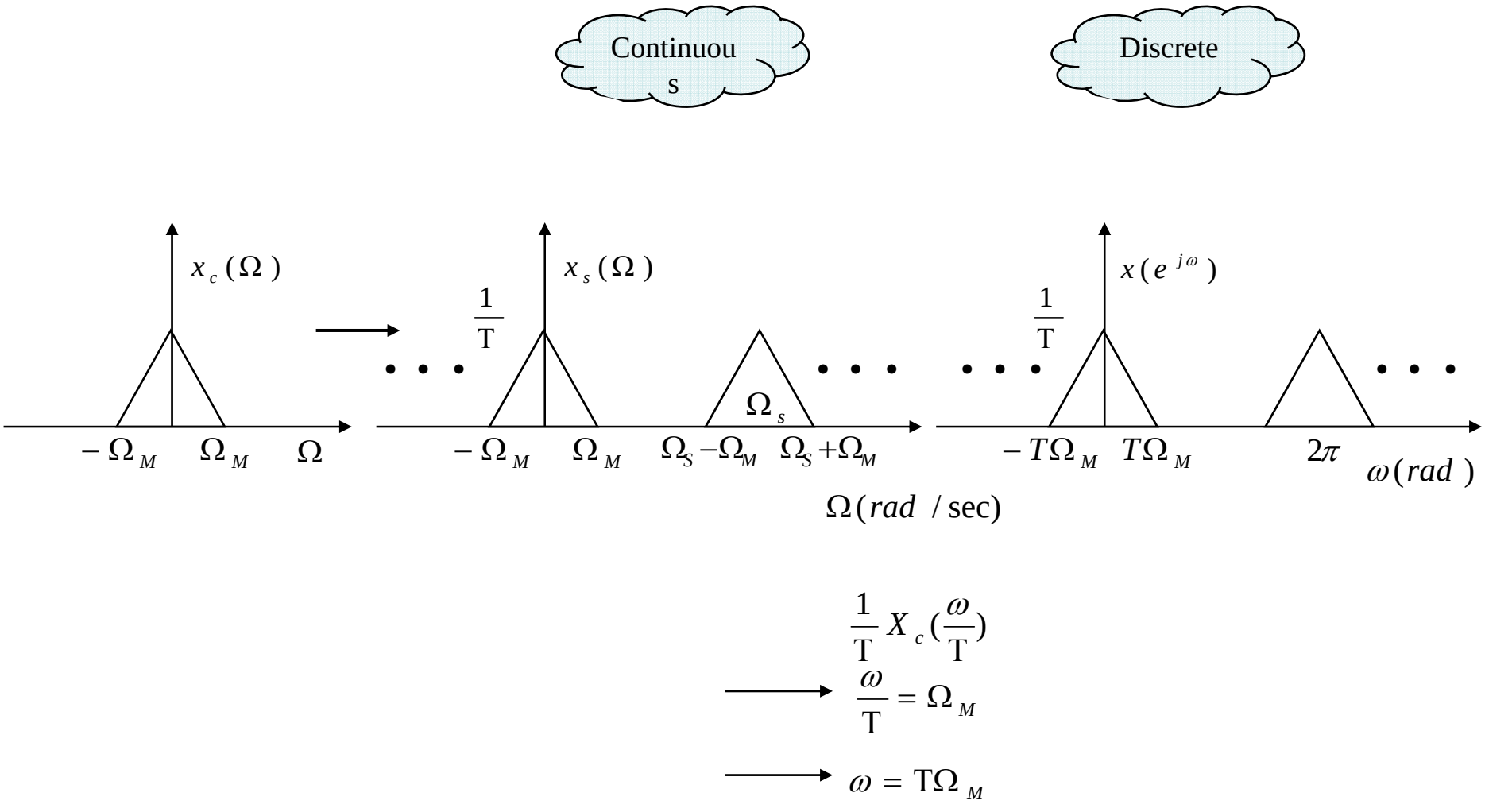

Continuous
Discrete
$x_c(\Omega)$
$x_s(\Omega)$
$x(e^{j\omega})$
$\frac{1}{T}$
$\frac{1}{T}$
Ω_s
$-\Omega_M$
Ω_M
Ω
$-\Omega_M$
Ω_M
$\Omega_S-\Omega_M$
$\Omega_S+\Omega_M$
$-T\Omega_M$
$T\Omega_M$
2π
$\omega(rad)$
$\Omega(rad/\sec)$
$\frac{1}{T}X_c(\frac{\omega}{T})$
$\frac{\omega}{T}=\Omega_M$
$\omega=T\Omega_M$

Figure 4.10

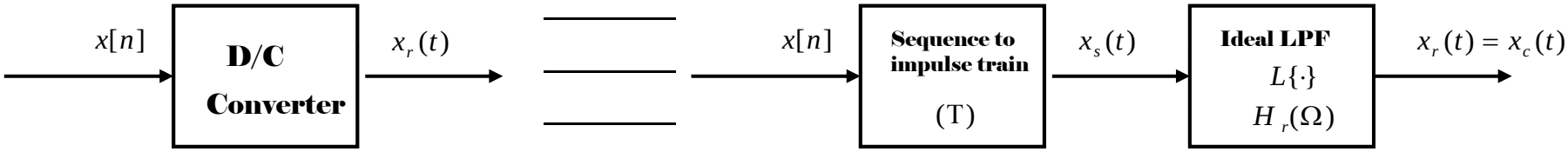

x[n]
D/C
Converter
x_r(t)
x[n]
Sequence to
impulse train
(T)
x_s(t)
Ideal LPF
L{·}
H_r(Ω)
x_r(t) = x_c(t)

Figure 4.11

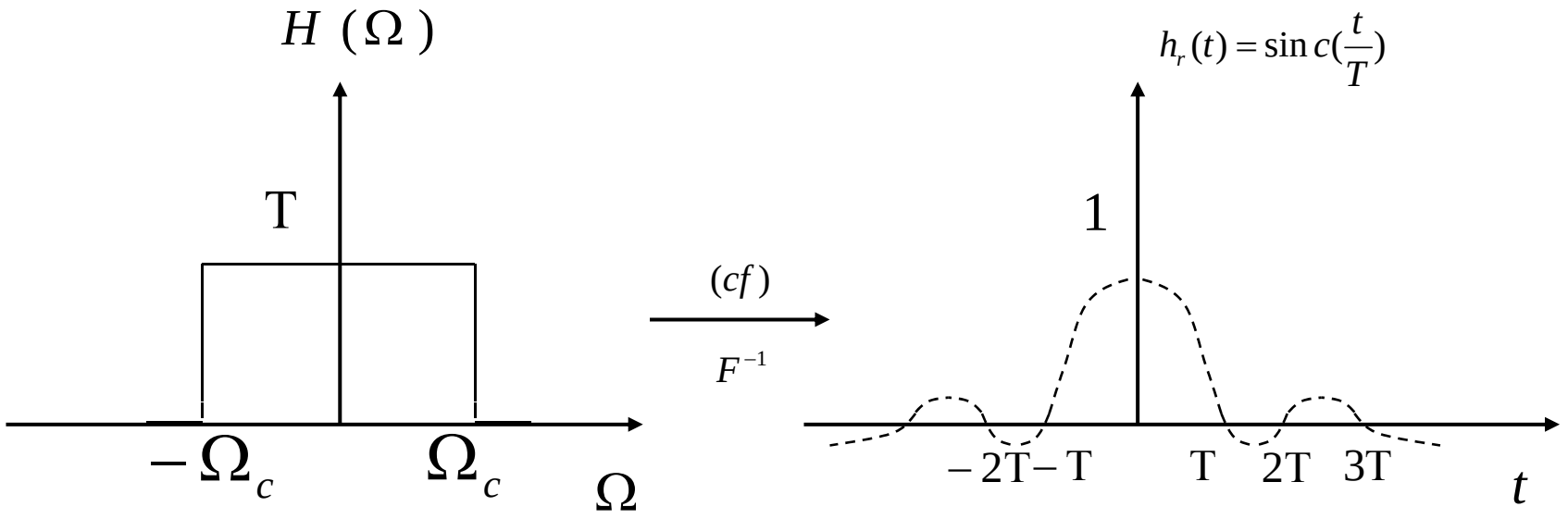


Figure 4.12

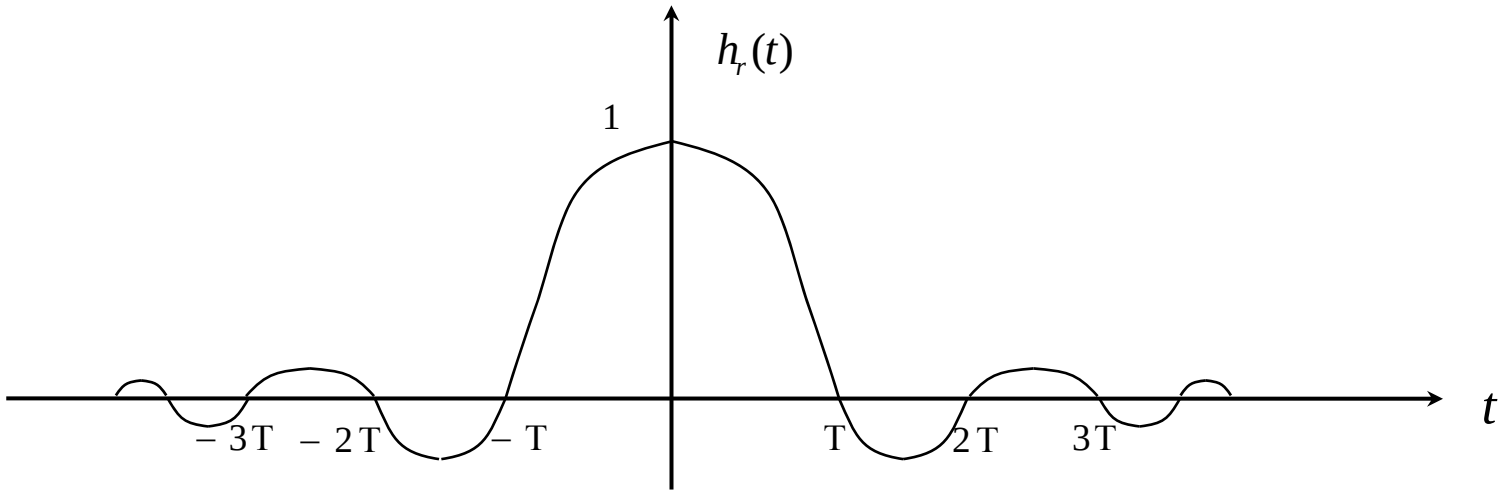


Figure 4.13

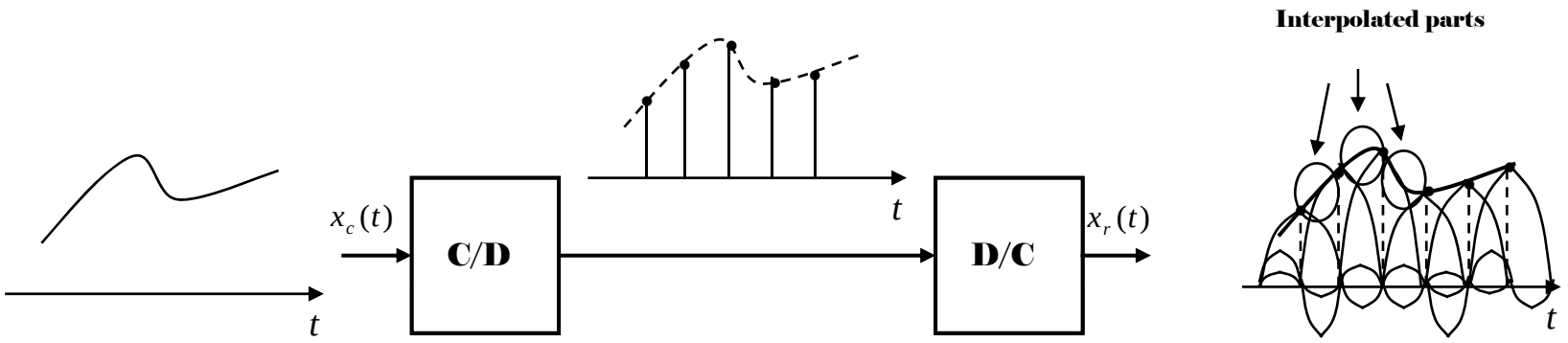


Figure 4.14

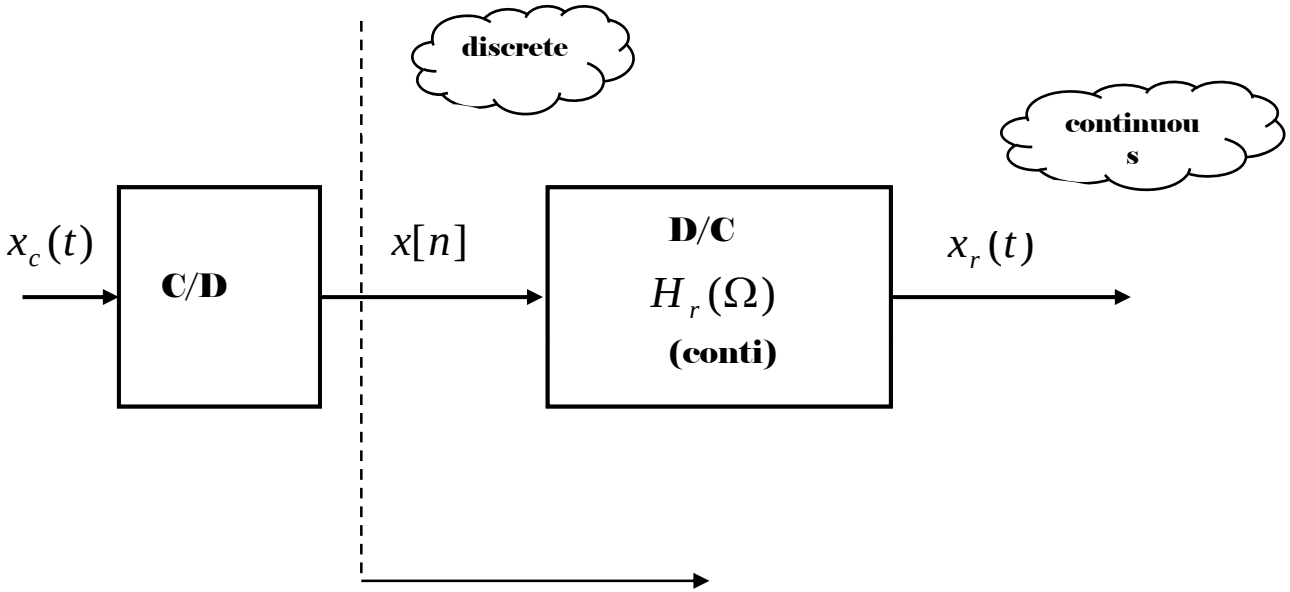


Figure 4.15

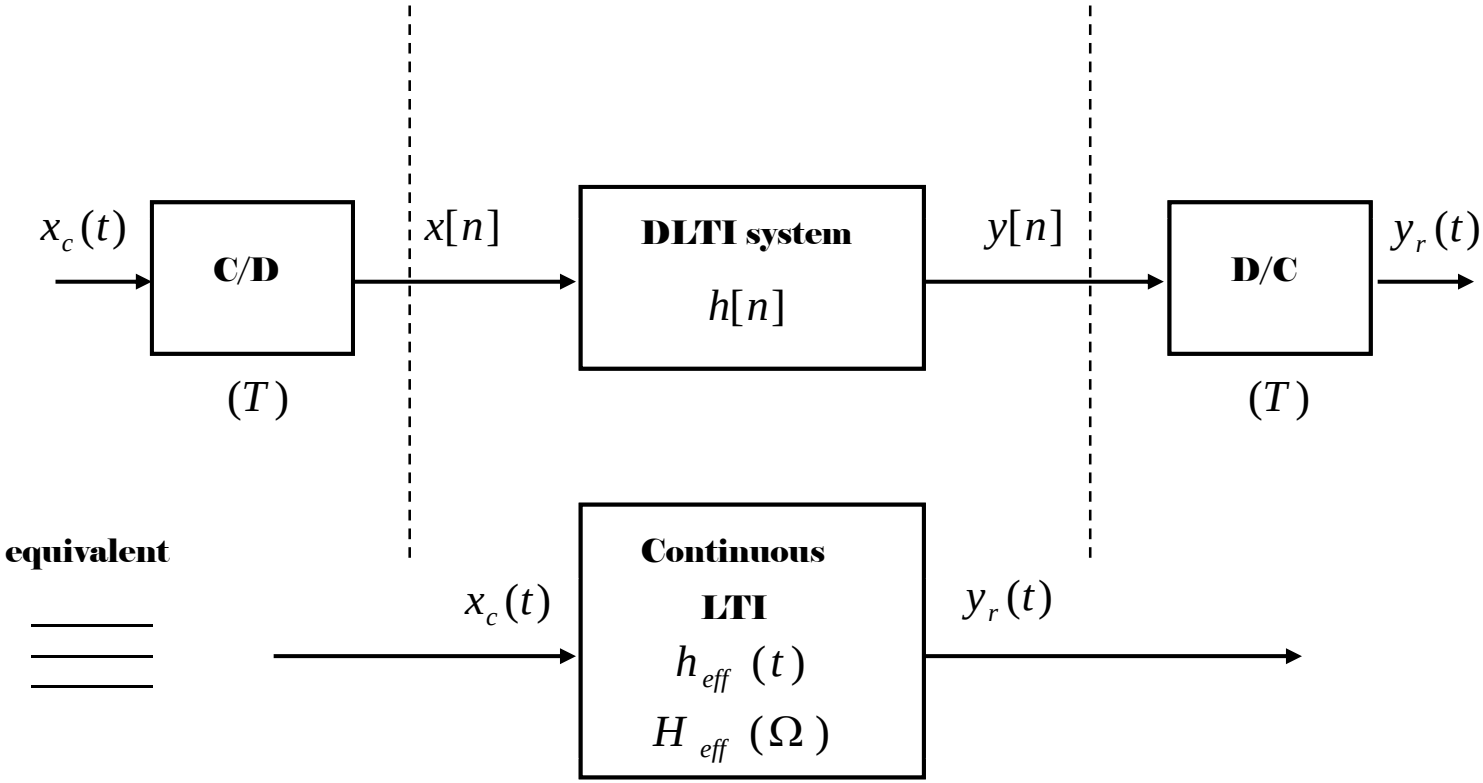

$x_c(t)$
C/D
(T)
$x[n]$
DLTI system
$h[n]$
$y[n]$
D/C
(T)
$y_r(t)$
equivalent
$x_c(t)$
Continuous
LTI
$h_{eff}(t)$
$H_{eff}(\Omega)$
$y_r(t)$

Figure 4.16

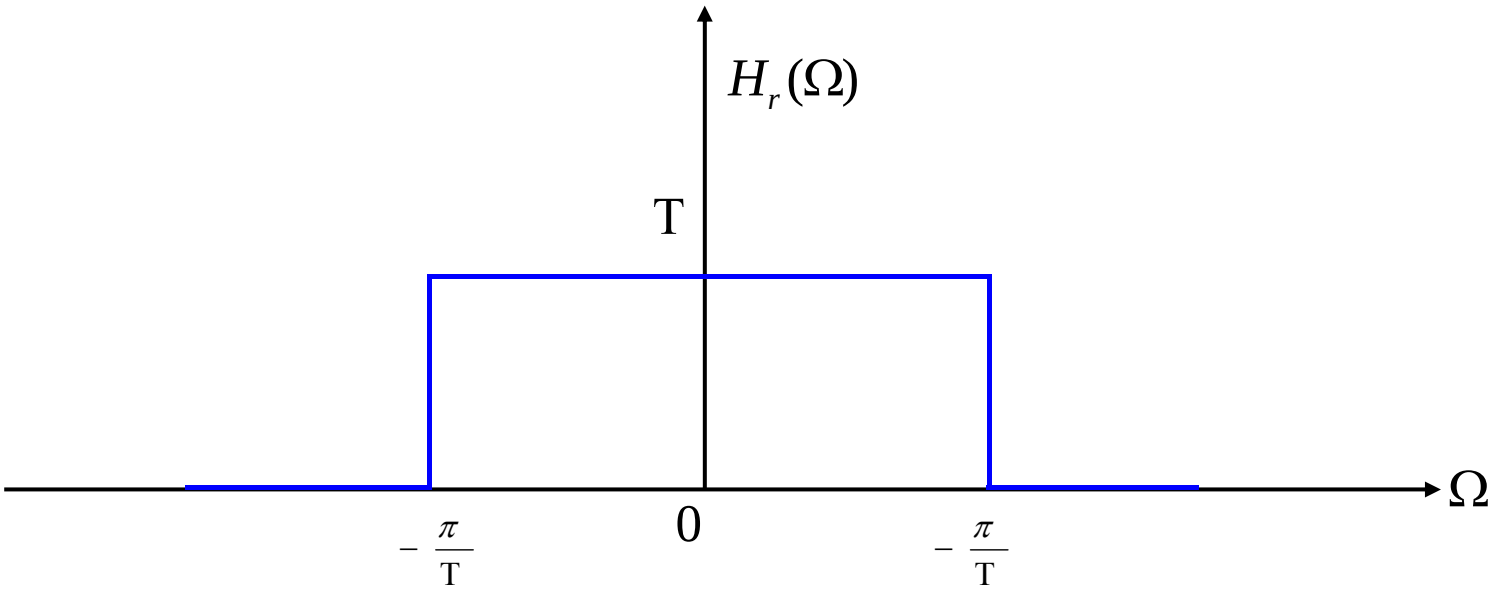


Figure 4.17

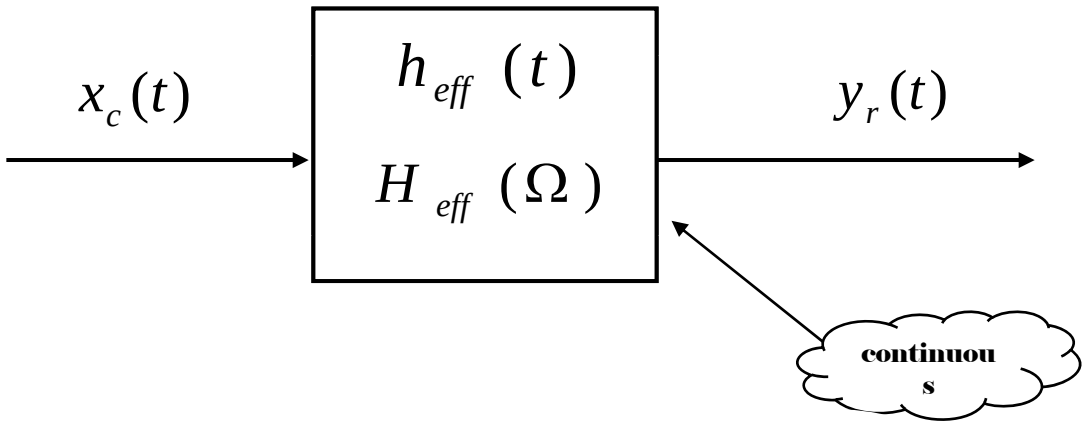


Figure 4.18

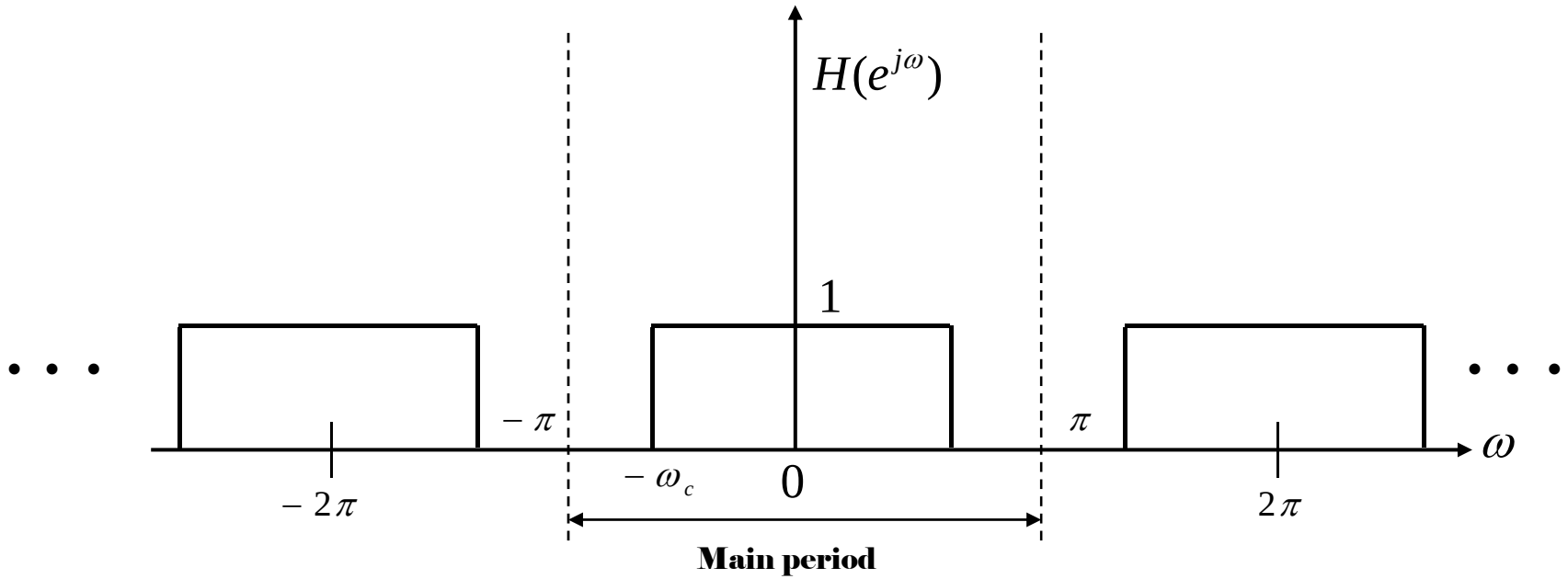


Figure 4.19

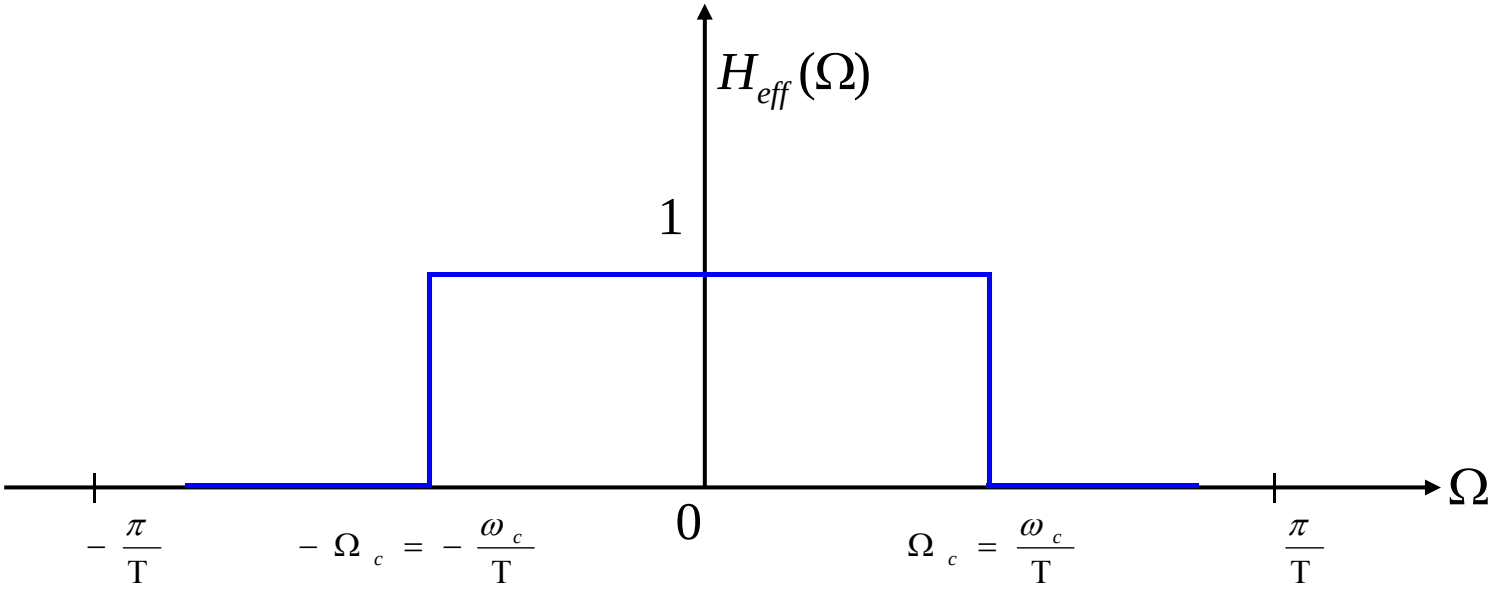

$H_{eff}(\Omega)$
1
0
Ω
$-\frac{\pi}{T}$
$-\Omega_c = -\frac{\omega_c}{T}$
$\Omega_c = \frac{\omega_c}{T}$
$\frac{\pi}{T}$

Figure 4.20

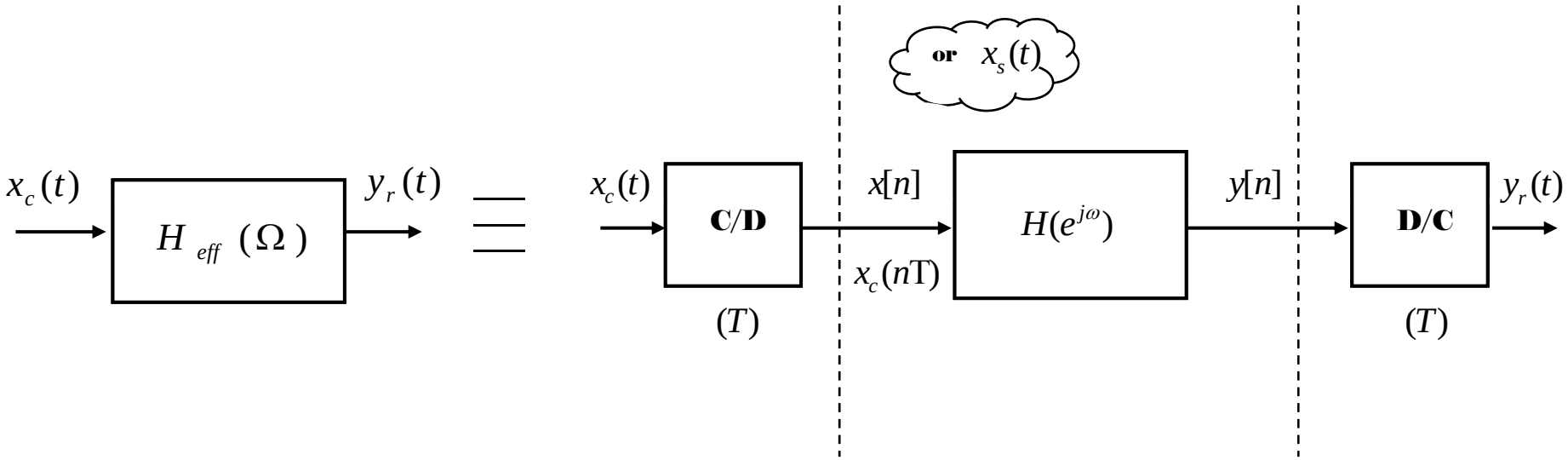


Figure 4.21

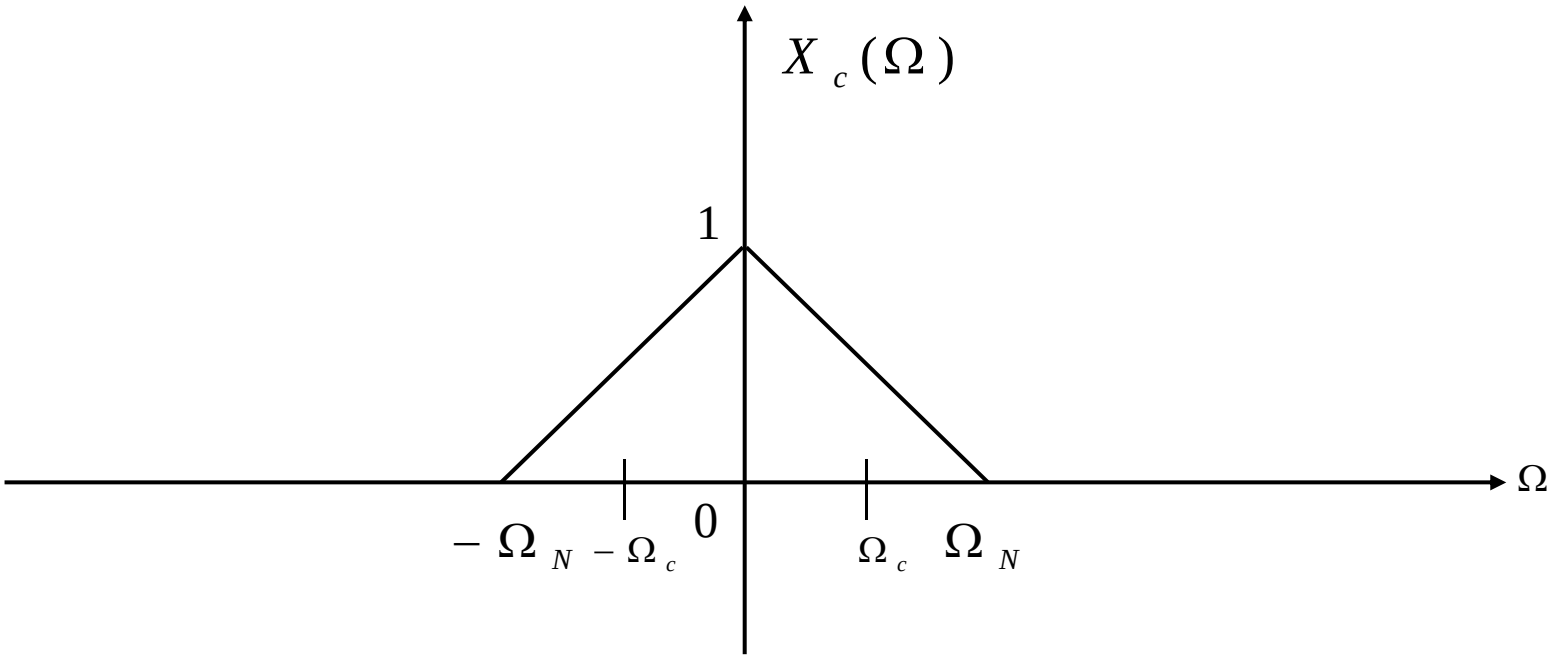


Figure 4.22

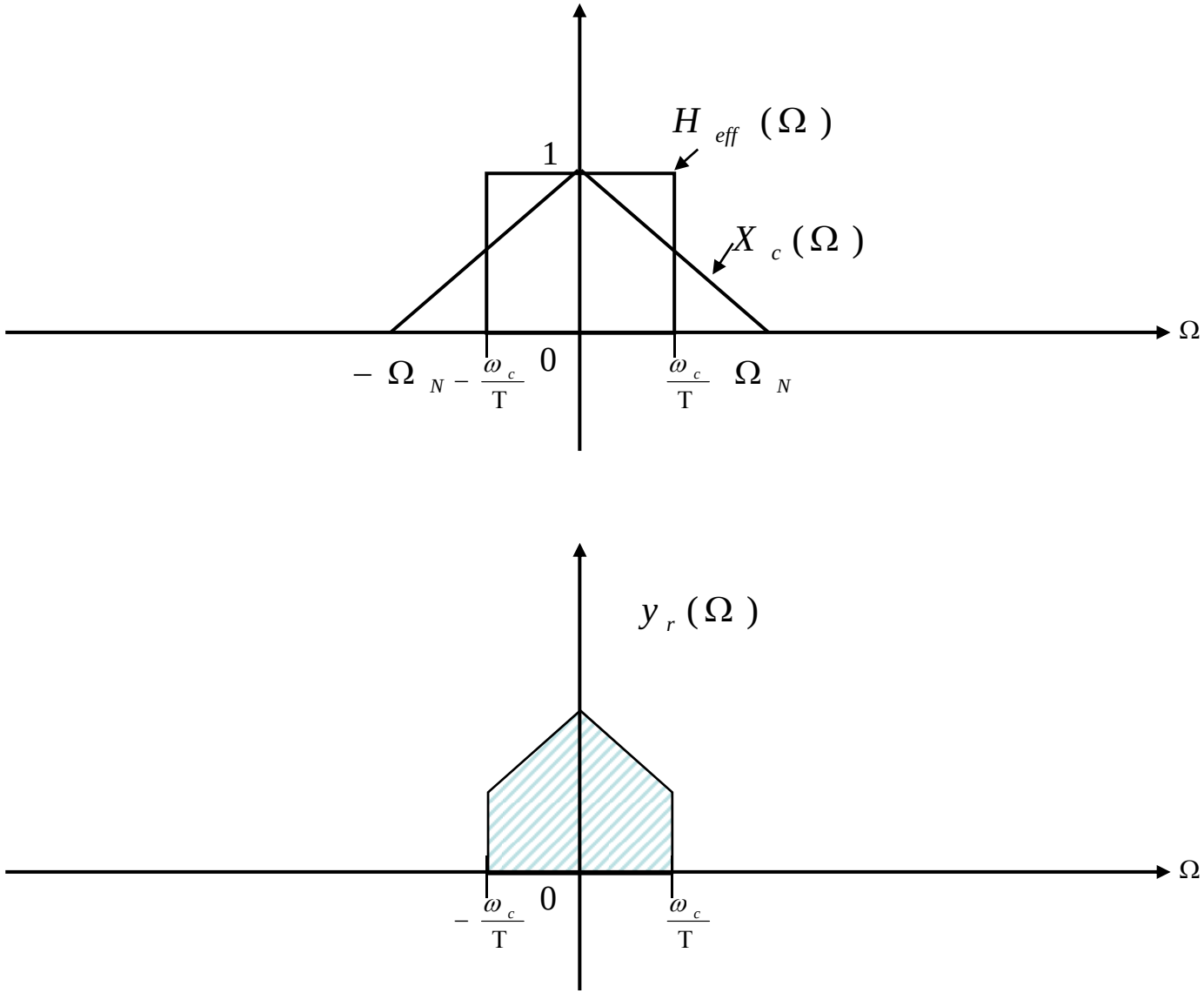

$H_{eff}(\Omega)$
1
$X_c(\Omega)$
Ω
$-\Omega_N$
$-\frac{\omega_c}{T}$
0
$\frac{\omega_c}{T}$
Ω_N
$y_r(\Omega)$
Ω
$-\frac{\omega_c}{T}$
0
$\frac{\omega_c}{T}$

Figure 4.23

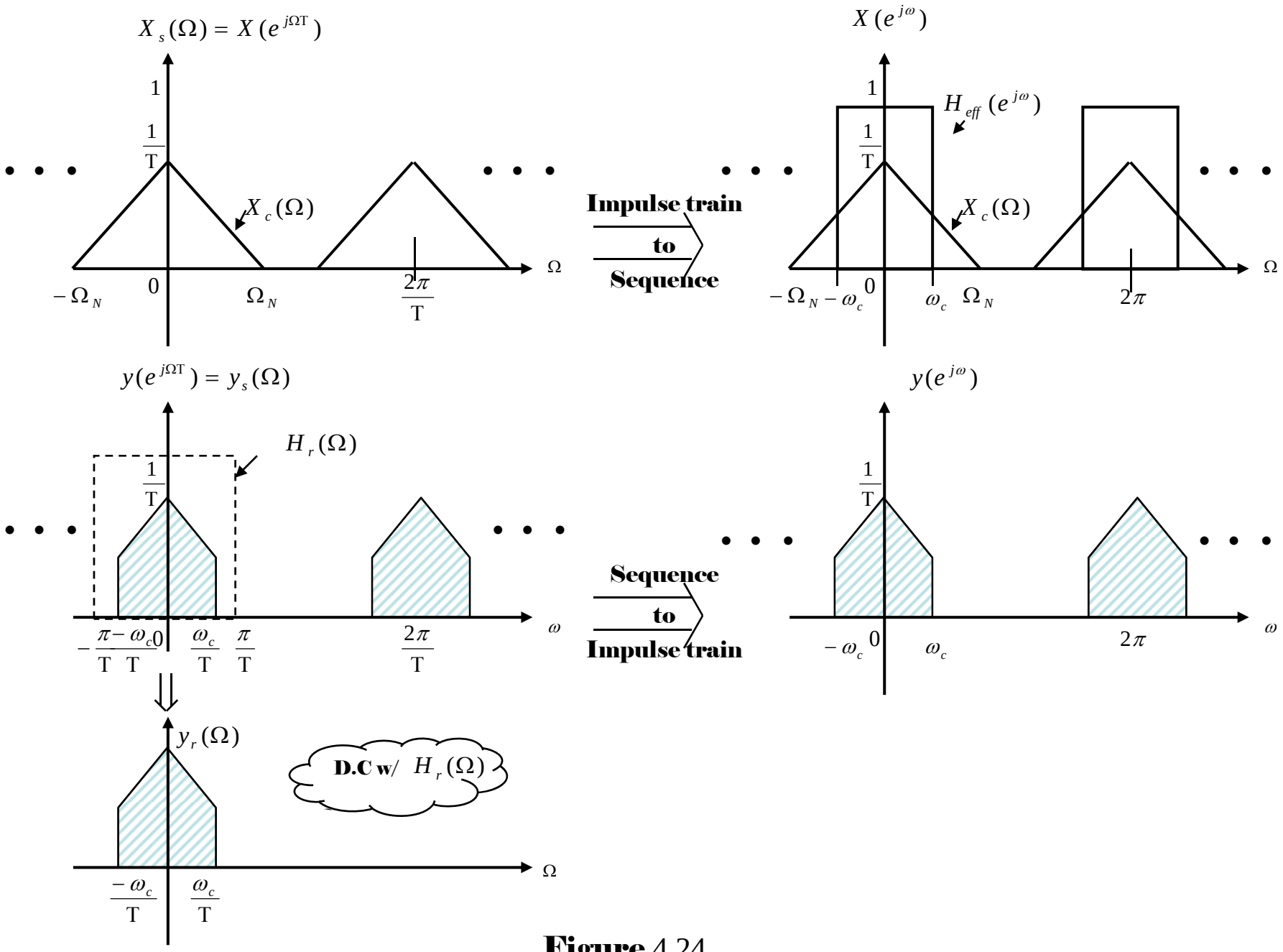

$X_s(\Omega) = X(e^{j\Omega T})$
1
$\frac{1}{T}$
$X_c(\Omega)$
$-\Omega_N$
0
Ω_N
$\frac{2\pi}{T}$
Ω
Impulse train
to
Sequence
$X(e^{j\omega})$
1
$H_{eff}(e^{j\omega})$
$\frac{1}{T}$
$X_c(\Omega)$
$-\Omega_N - \omega_c$
0
ω_c
Ω_N
2π
Ω
$y(e^{j\Omega T}) = y_s(\Omega)$
$H_r(\Omega)$
$\frac{1}{T}$
$-\frac{\pi - \omega_c}{T}$
0
$\frac{\omega_c}{T}$
$\frac{\pi}{T}$
$\frac{2\pi}{T}$
ω
$y_r(\Omega)$
D.C w/ $H_r(\Omega)$
$\frac{-\omega_c}{T}$
$\frac{\omega_c}{T}$
Ω
Sequence
to
Impulse train
$y(e^{j\omega})$
$\frac{1}{T}$
$-\omega_c$
0
ω_c
2π
ω

Figure 4.24

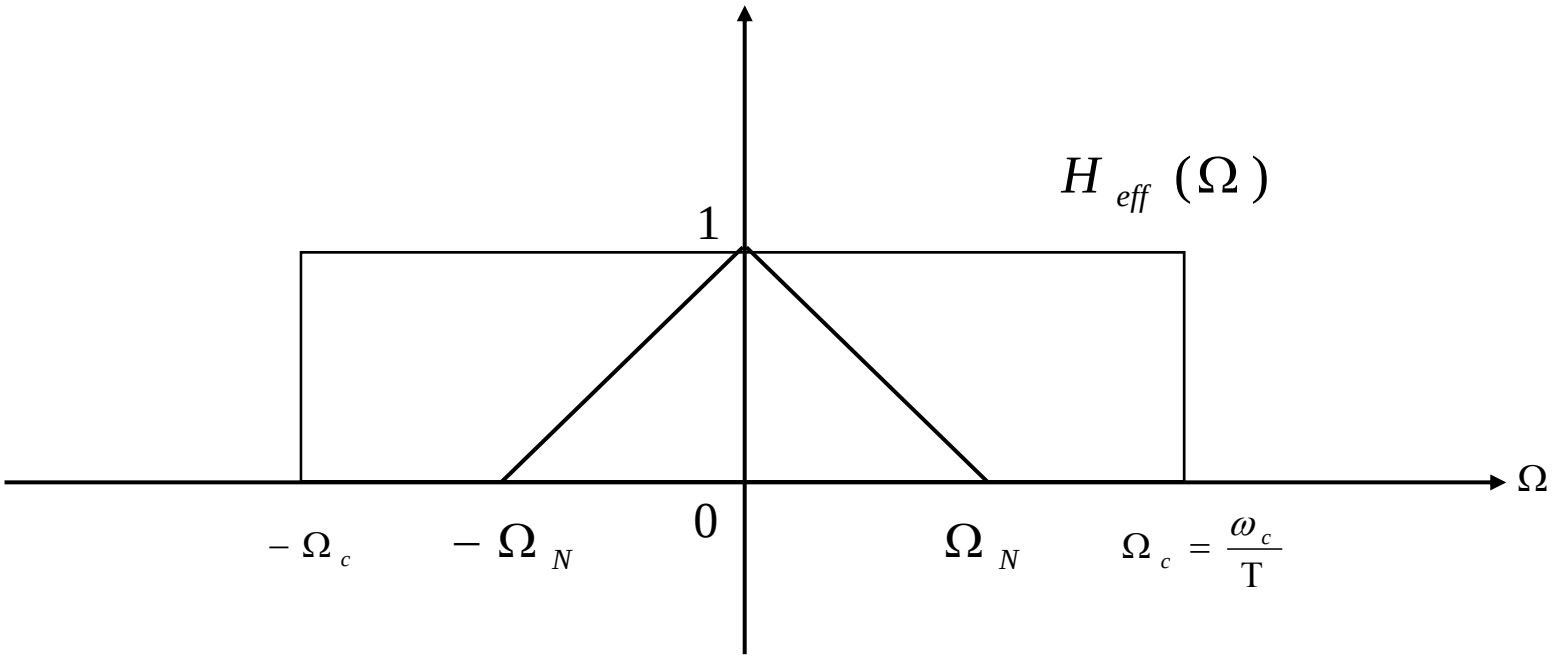

$H_{eff}(\Omega)$
1
0
Ω
$-\Omega_c$
$-\Omega_N$
Ω_N
$\Omega_c = \frac{\omega_c}{T}$

Figure 4.25

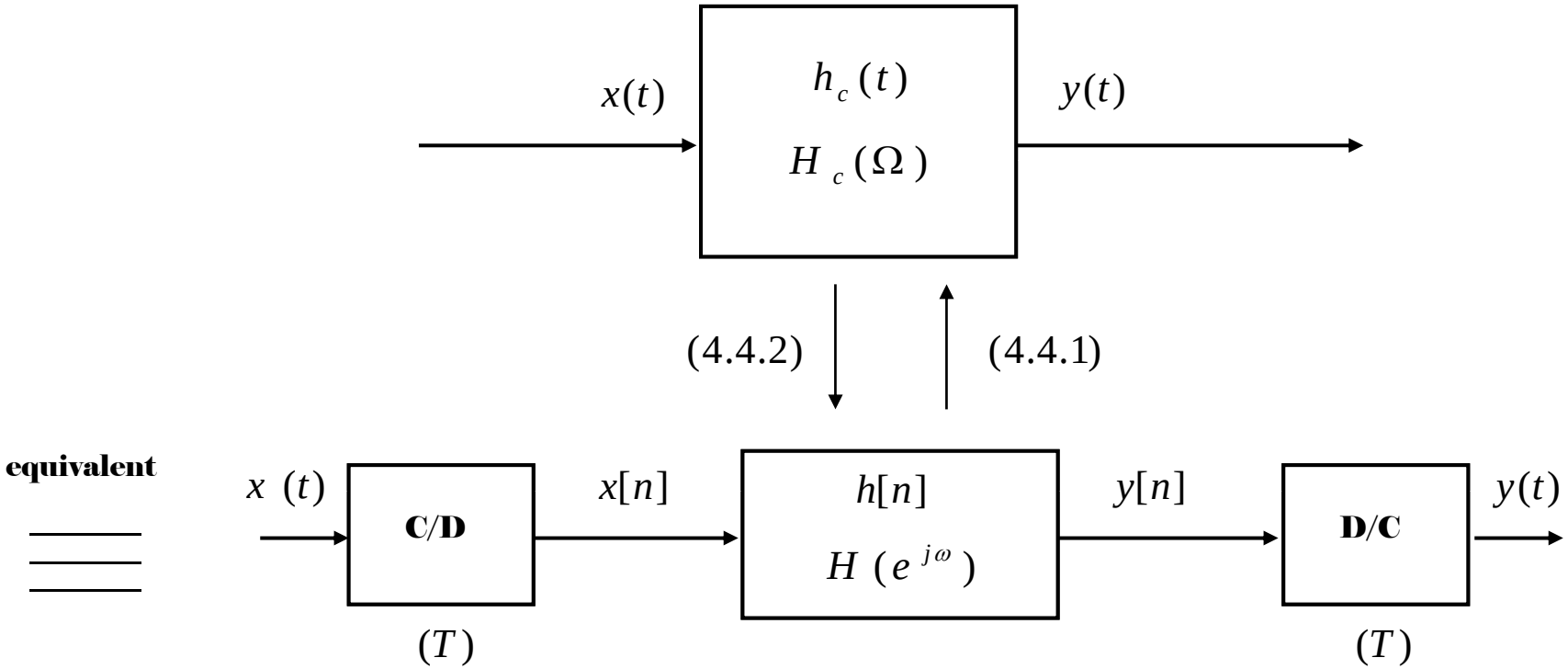

$x(t)$
$h_c(t)$
$H_c(\Omega)$
$y(t)$
(4.4.2)
(4.4.1)
equivalent
$x(t)$
C/D
$x[n]$
$h[n]$
$H(e^{j\omega})$
$y[n]$
D/C
$y(t)$
(T)
(T)

Figure 4.26

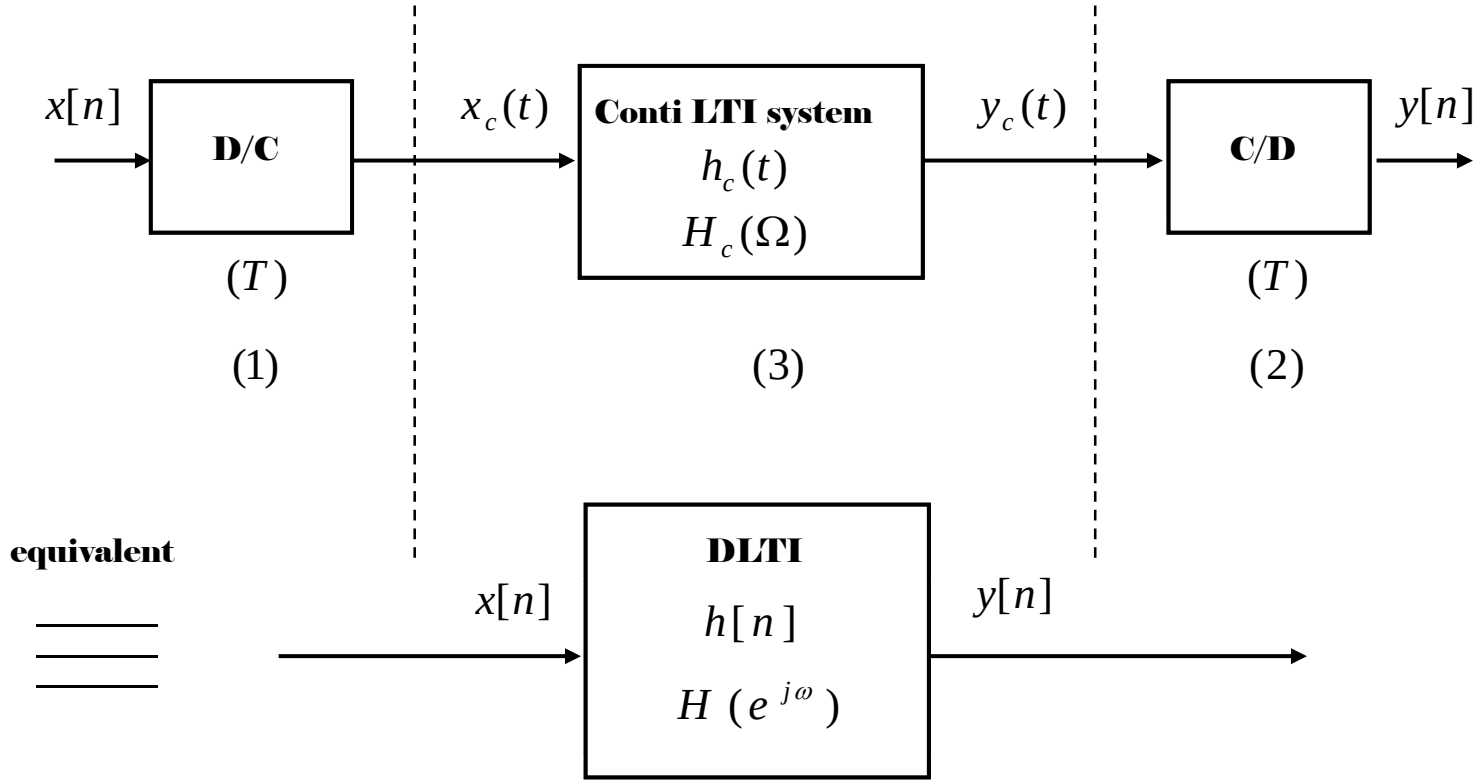

x[n]
D/C
(T)
(1)
x_c(t)
Conti LTI system
h_c(t)
H_c(Ω)
(3)
y_c(t)
C/D
(T)
(2)
y[n]
equivalent
x[n]
DLTI
h[n]
H (e^{jω})
y[n]

Figure 4.27

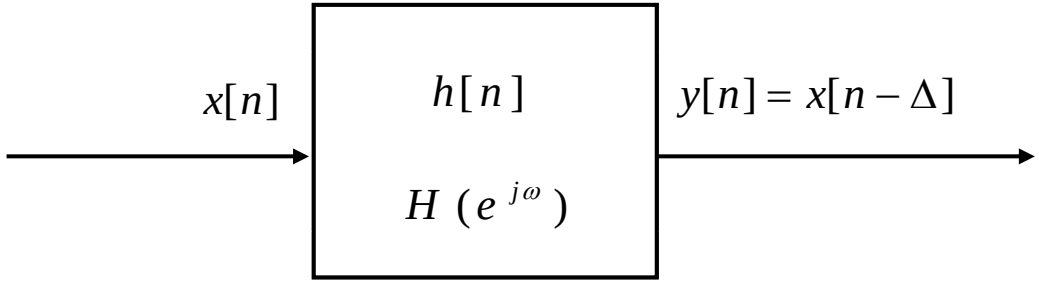


Figure 4.28

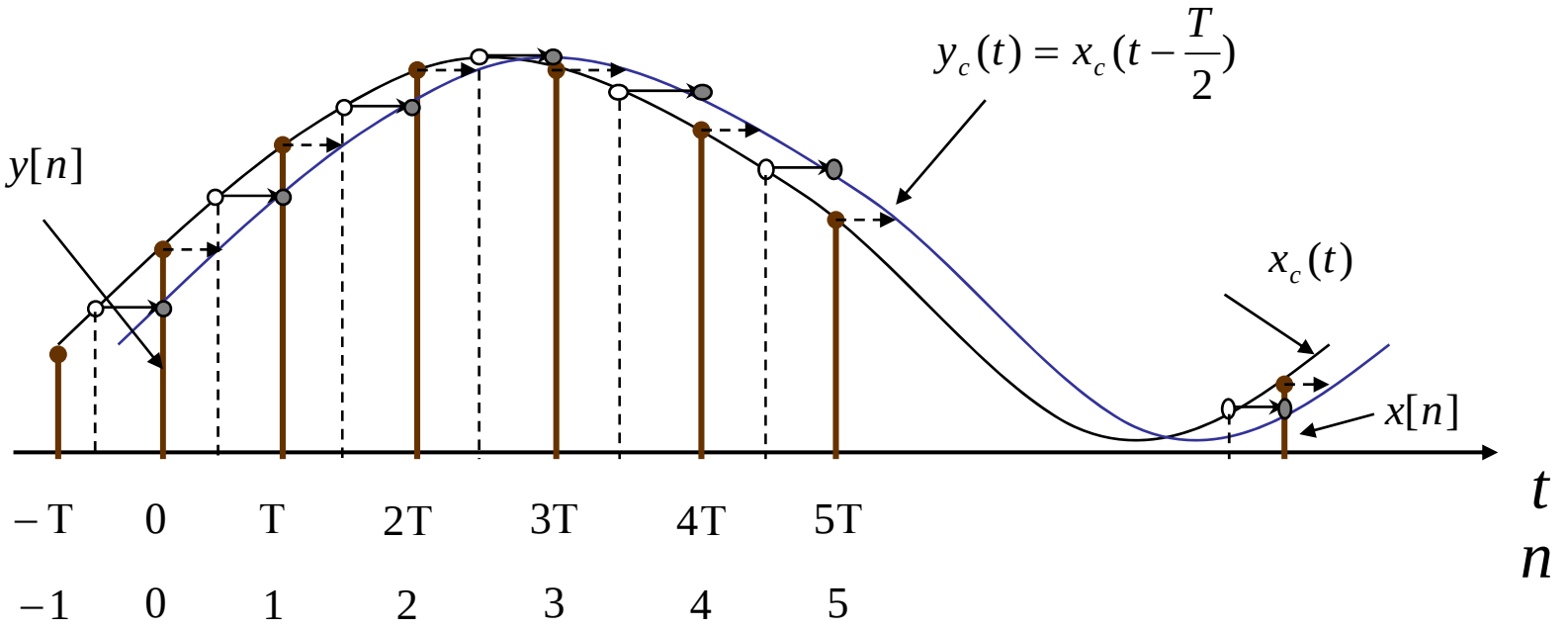


Figure 4.29

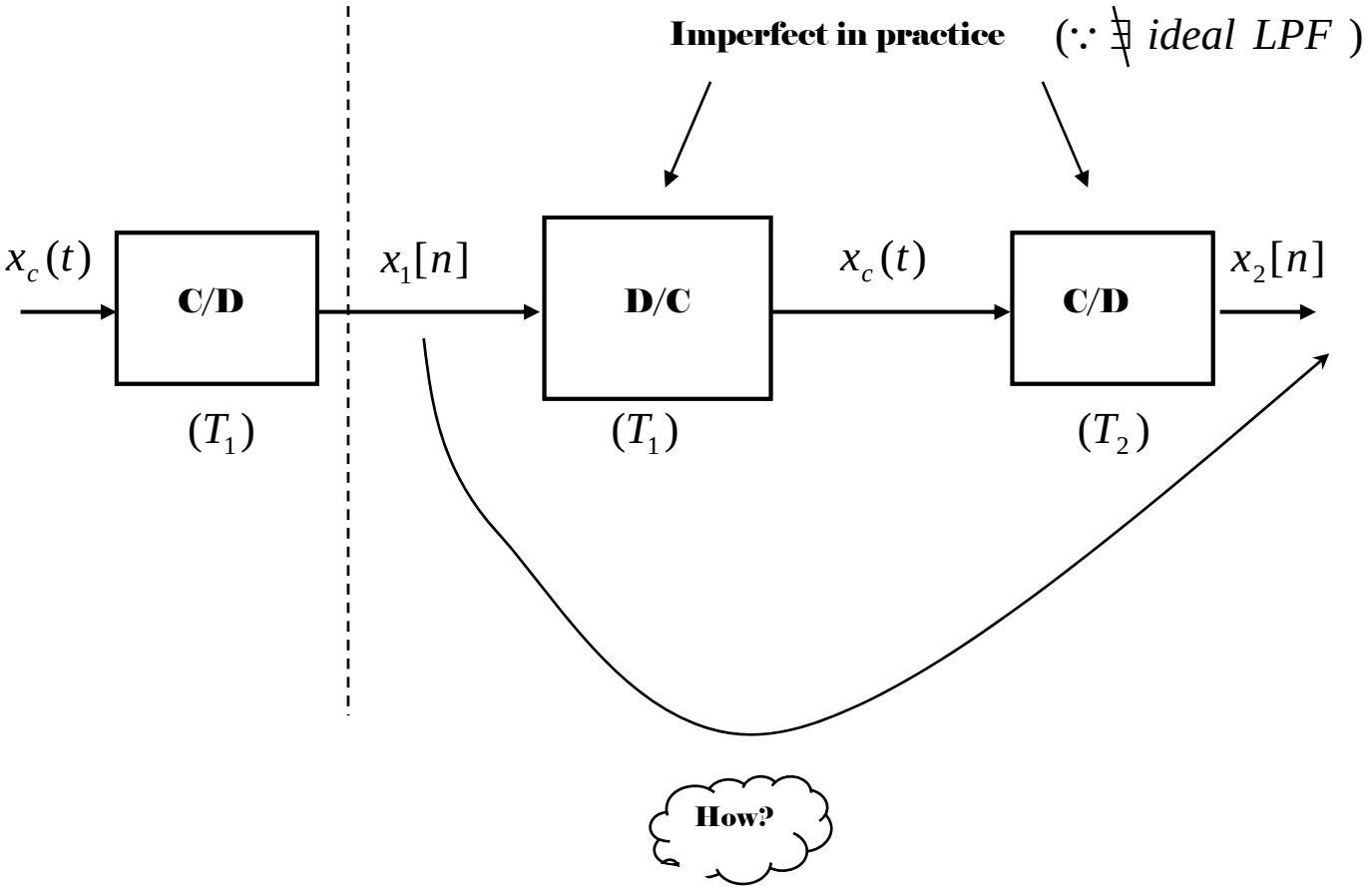

Imperfect in practice
$(\because \nexists$ ideal LPF $)$
$x_c(t)$
C/D
(T_1)
$x_1[n]$
D/C
(T_1)
$x_c(t)$
C/D
(T_2)
$x_2[n]$
How?

Figure 4.30

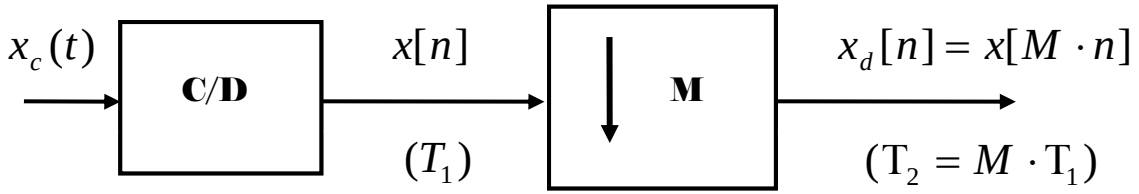


Figure 4.31

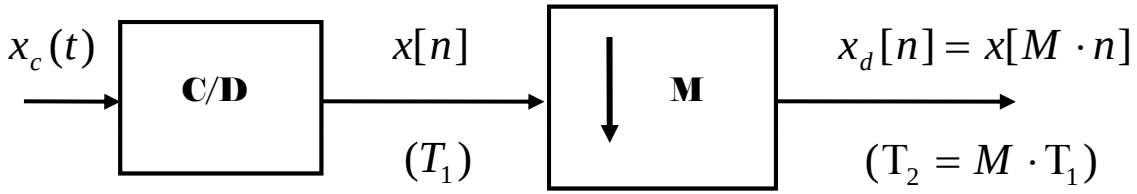


Figure 4.31

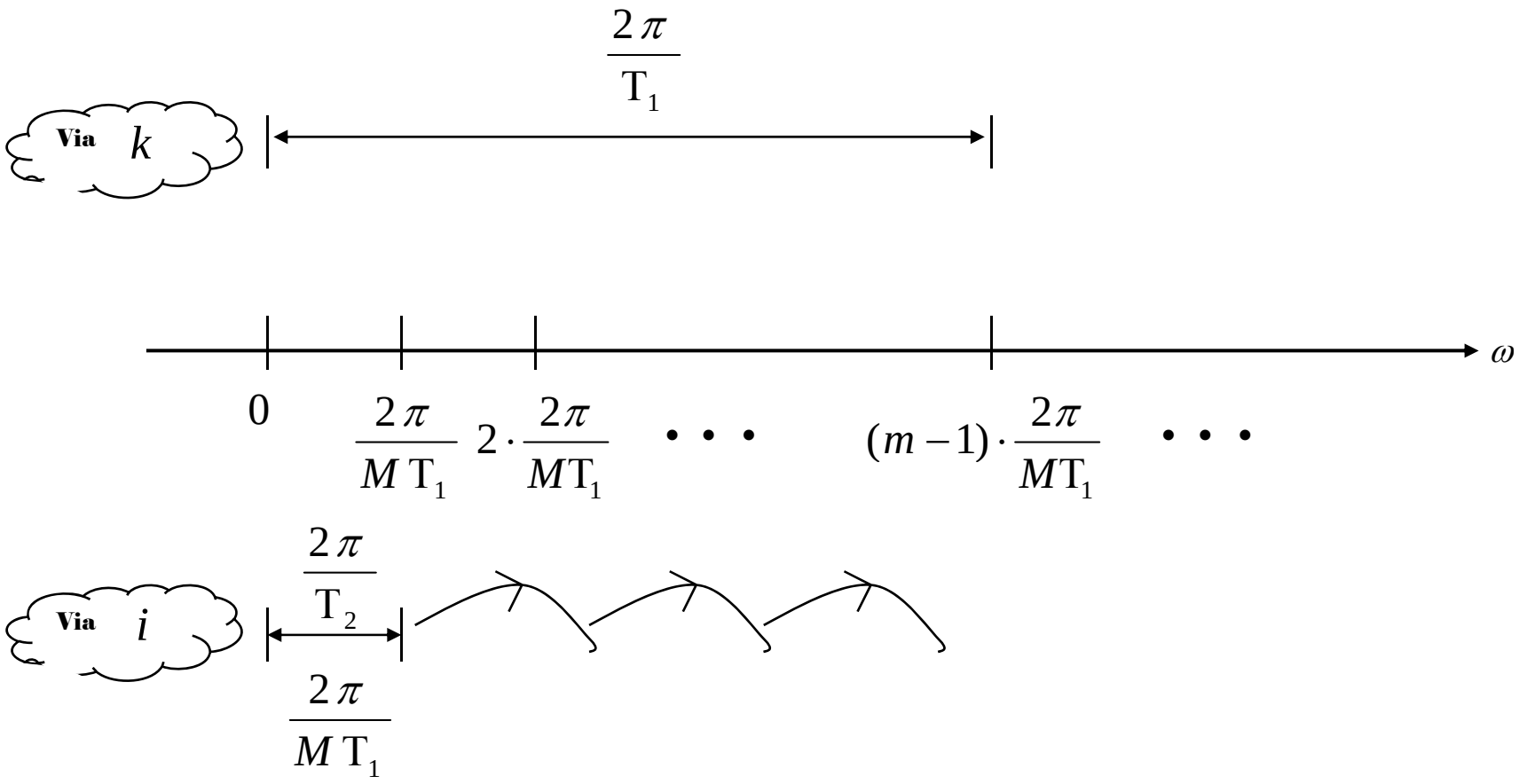

$\frac{2\pi}{T_1}$
Via k
ω
0
$\frac{2\pi}{MT_1}$
$2\cdot\frac{2\pi}{MT_1}$
• • •
$(m-1)\cdot\frac{2\pi}{MT_1}$
• • •
$\frac{2\pi}{T_2}$
Via i
$\frac{2\pi}{MT_1}$

Figure 4.32

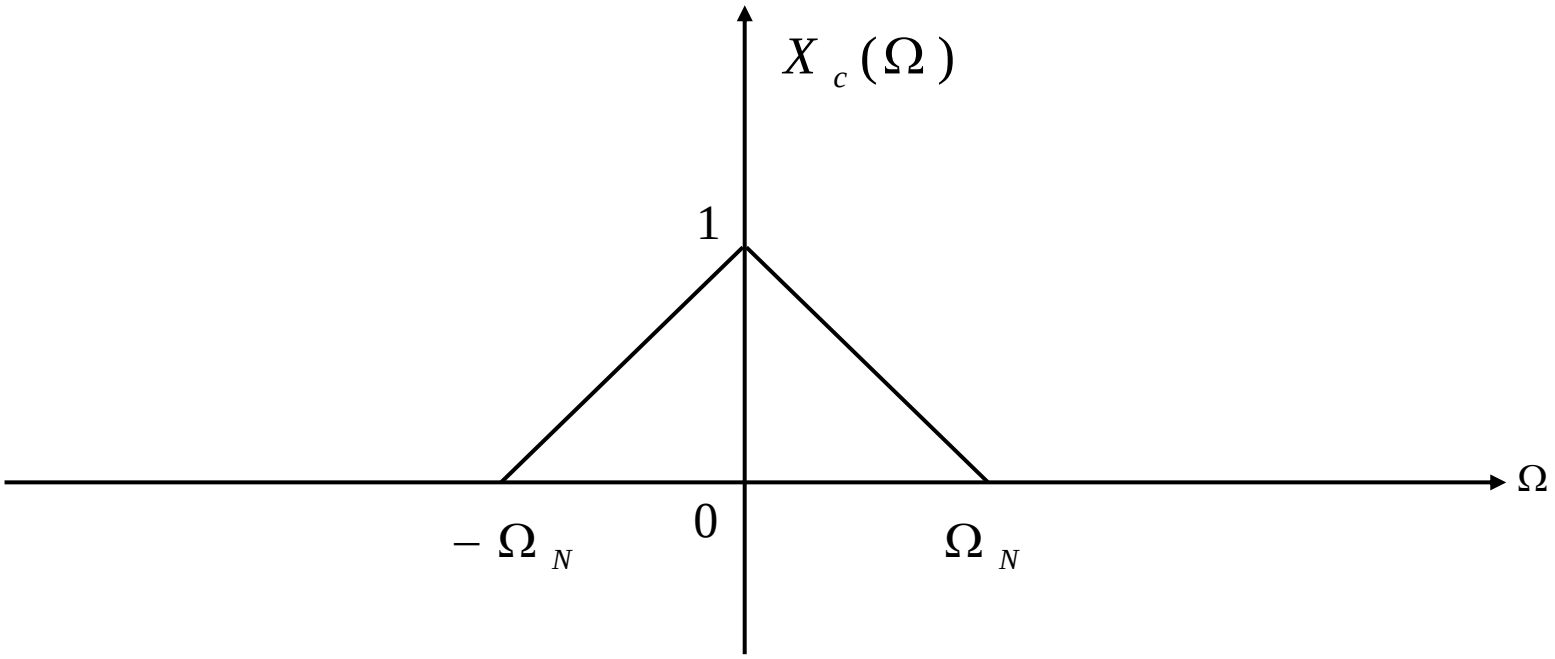


Figure 4.33

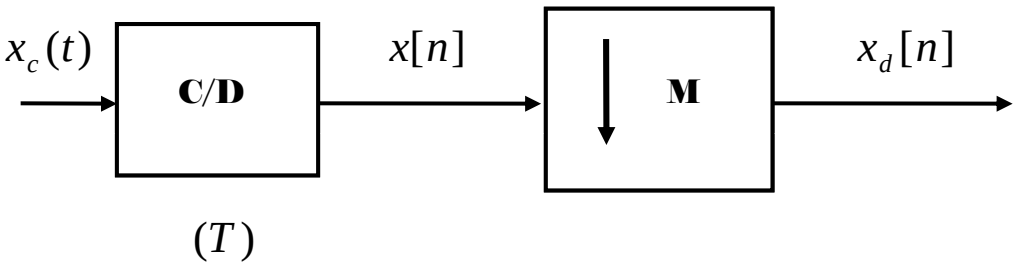


Figure 4.34

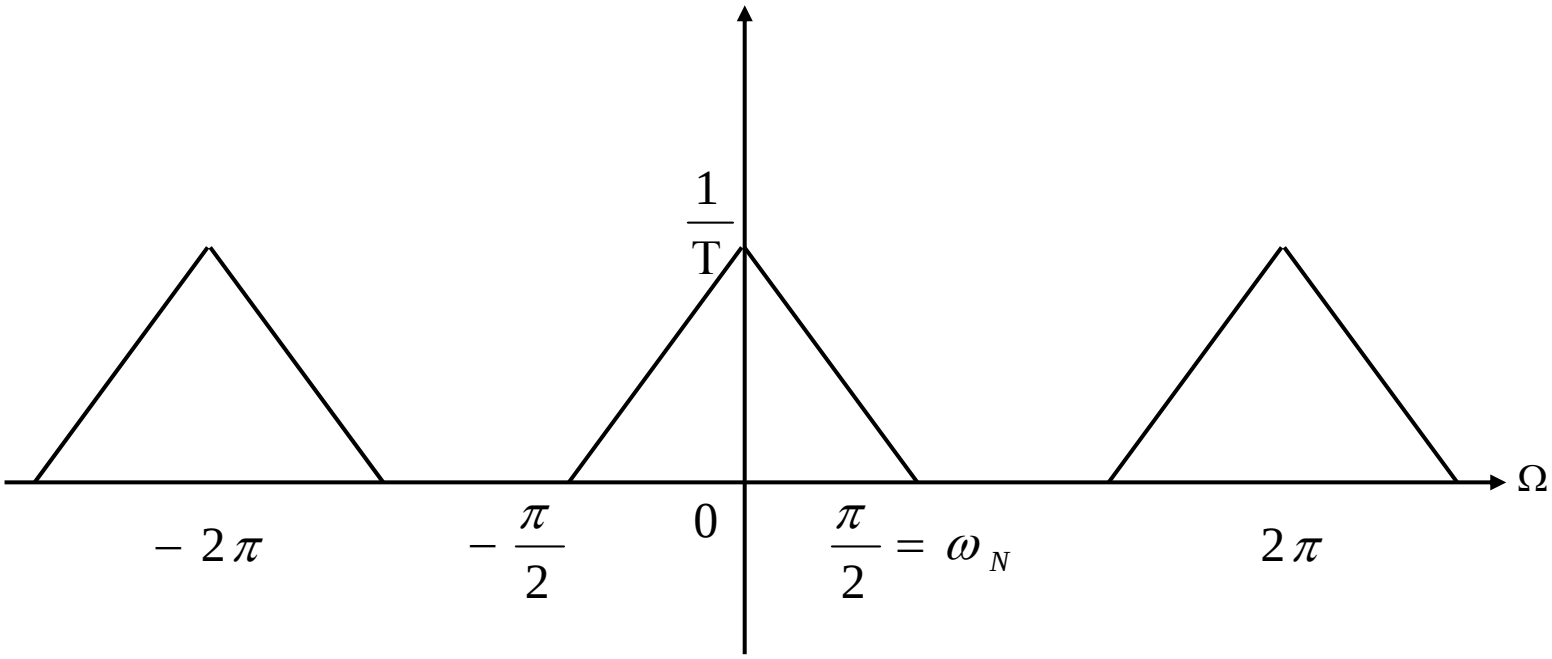


Figure 4.35

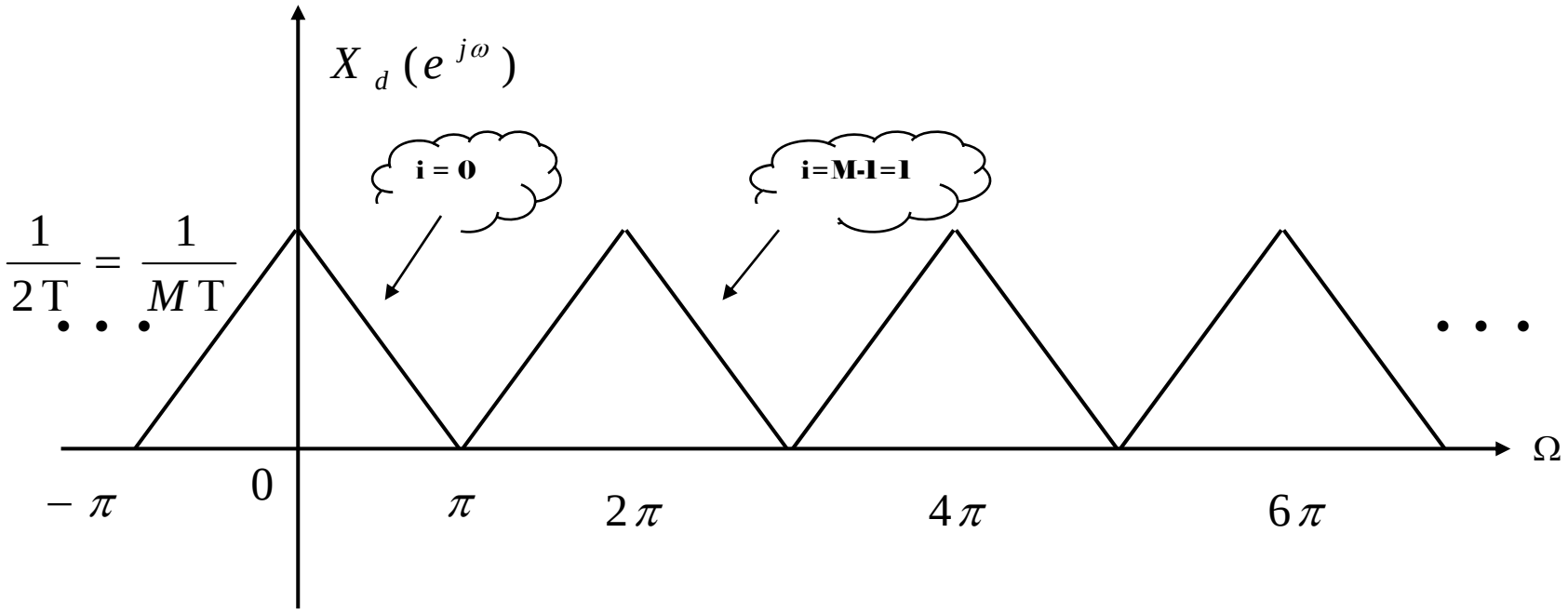


Figure 4.36

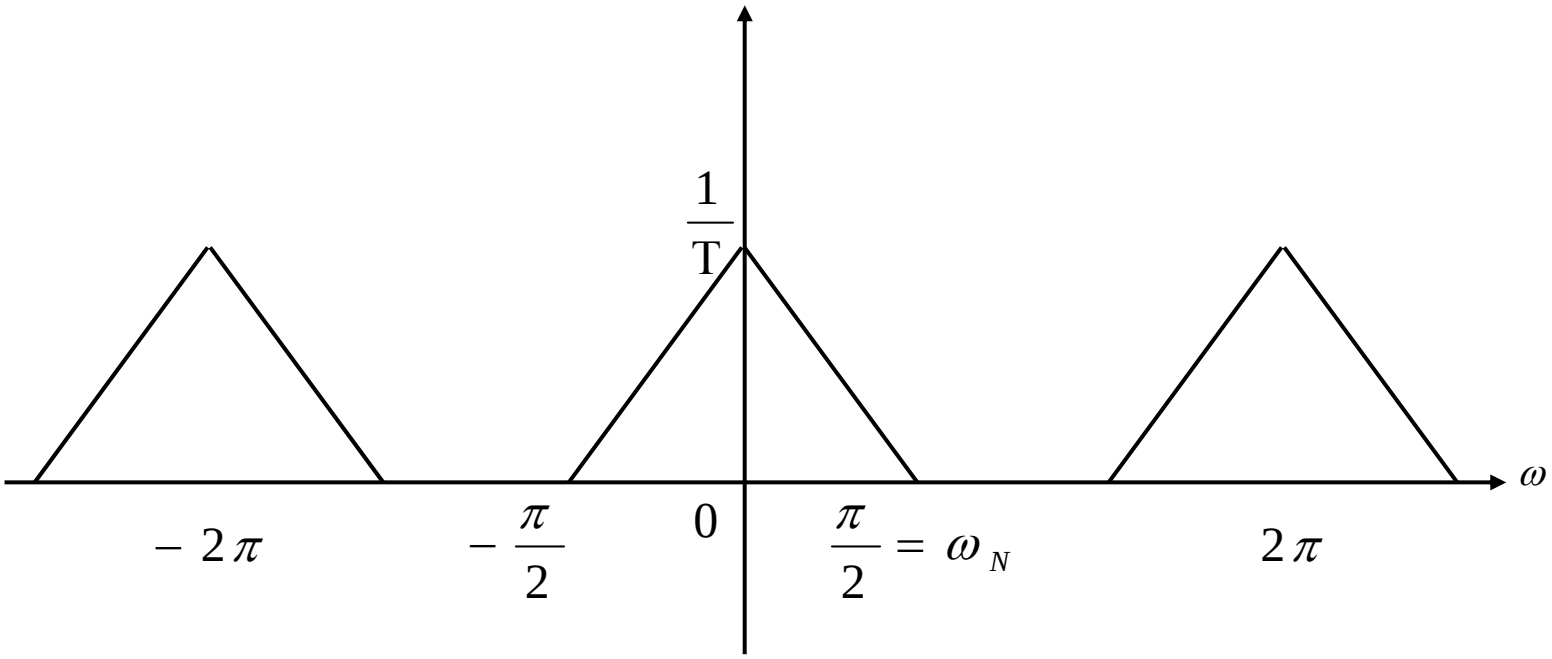


Figure 4.37

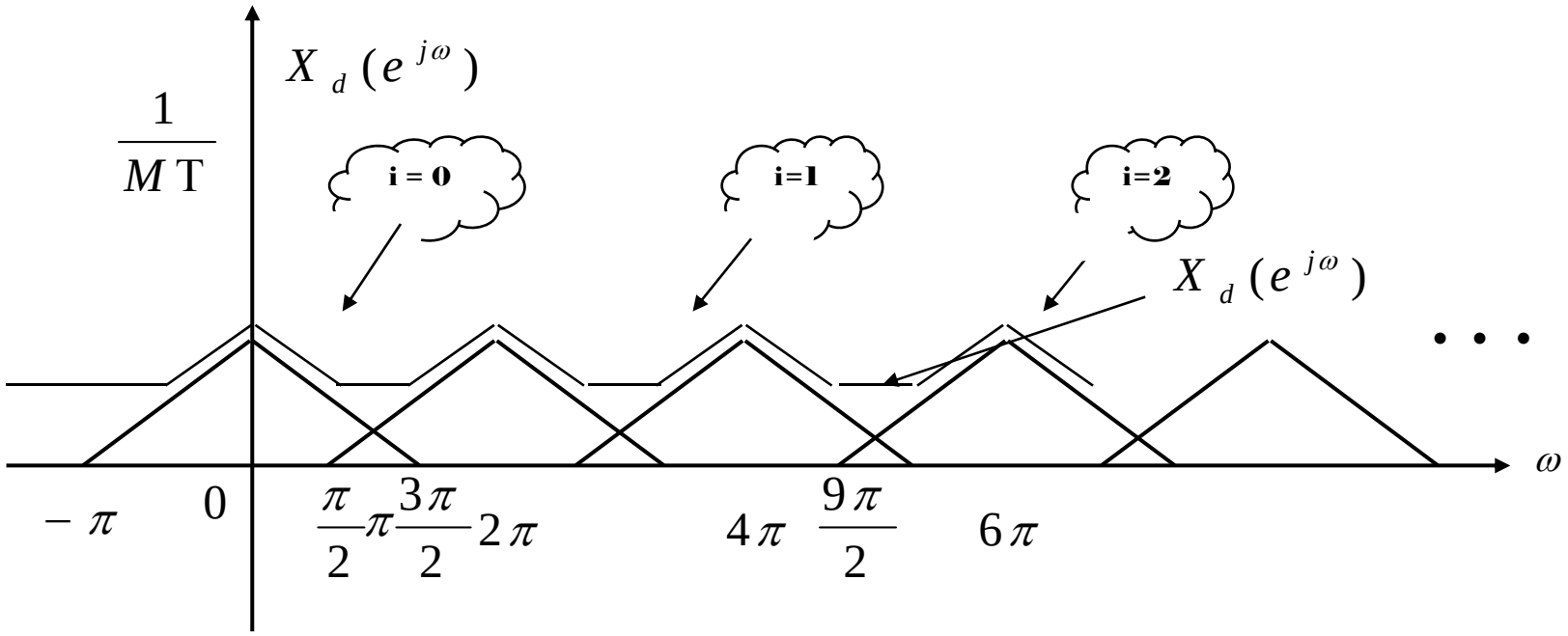

$X_d(e^{j\omega})$
$\frac{1}{MT}$
i = 0
i=1
i=2
$X_d(e^{j\omega})$
ω
$-\pi$
0
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
4π
$\frac{9\pi}{2}$
6π

Figure 4.38

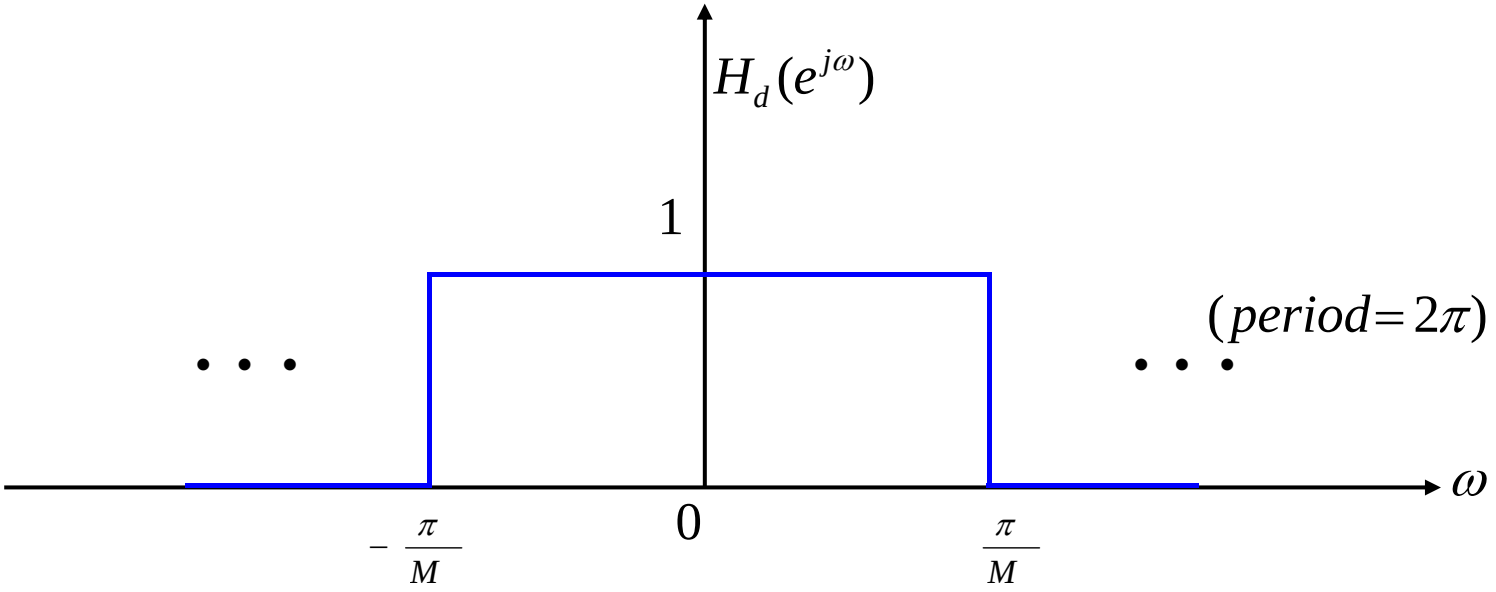


Figure 4.39

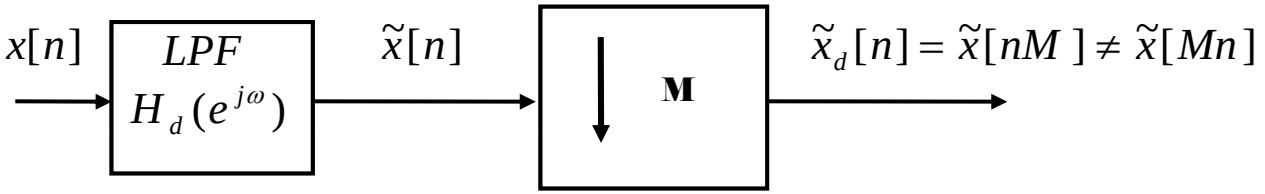

$x[n]$
LPF
$H_d(e^{j\omega})$
$\tilde{x}[n]$
M
$\tilde{x}_d[n] = \tilde{x}[nM] \neq \tilde{x}[Mn]$
" pre − filtering "

Figure 4.40

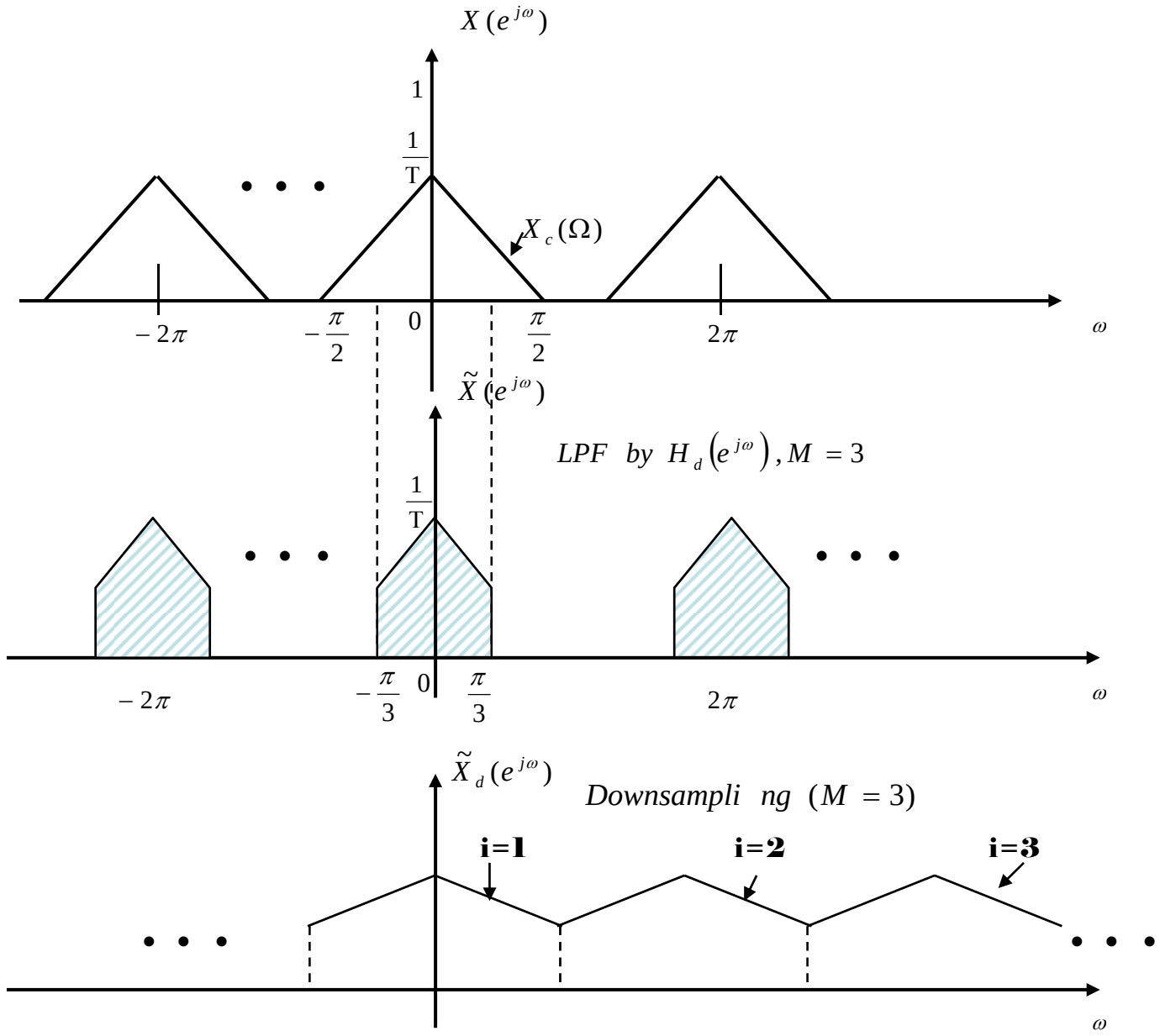
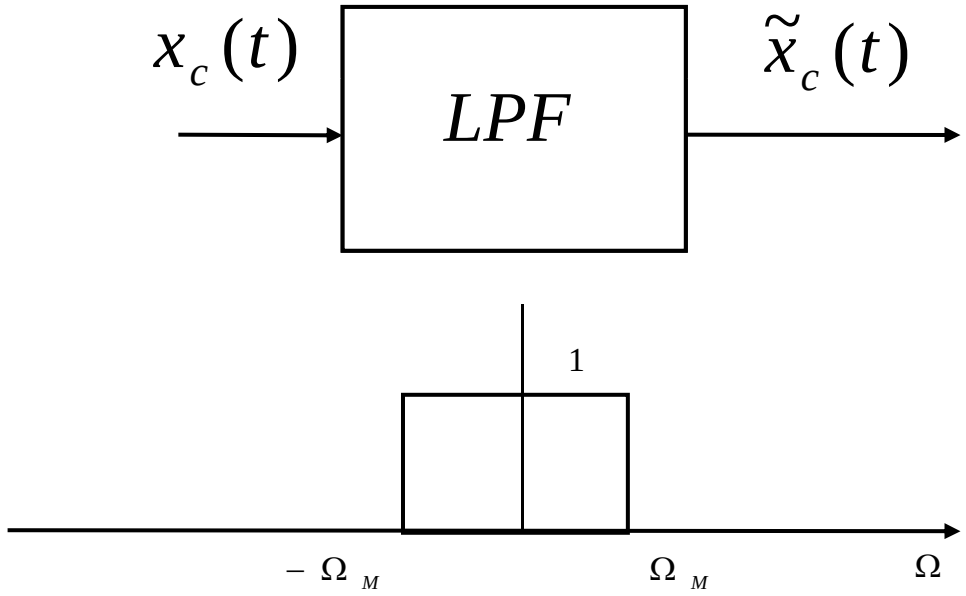

$X(e^{j\omega})$
1
$\frac{1}{T}$
$X_c(\Omega)$
-2π
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
2π
ω
$\tilde{X}(e^{j\omega})$
LPF by $H_d(e^{j\omega}), M = 3$
$\frac{1}{T}$
-2π
$-\frac{\pi}{3}$
0
$\frac{\pi}{3}$
2π
ω
$\tilde{X}_d(e^{j\omega})$
Downsampli ng $(M = 3)$
i=1
i=2
i=3
ω

Figure 4.41



$$\Omega_M = \frac{\pi}{M} \cdot \frac{1}{\mathrm{T}}$$

$$(\omega = \Omega \mathrm{T})$$

Figure 4.42

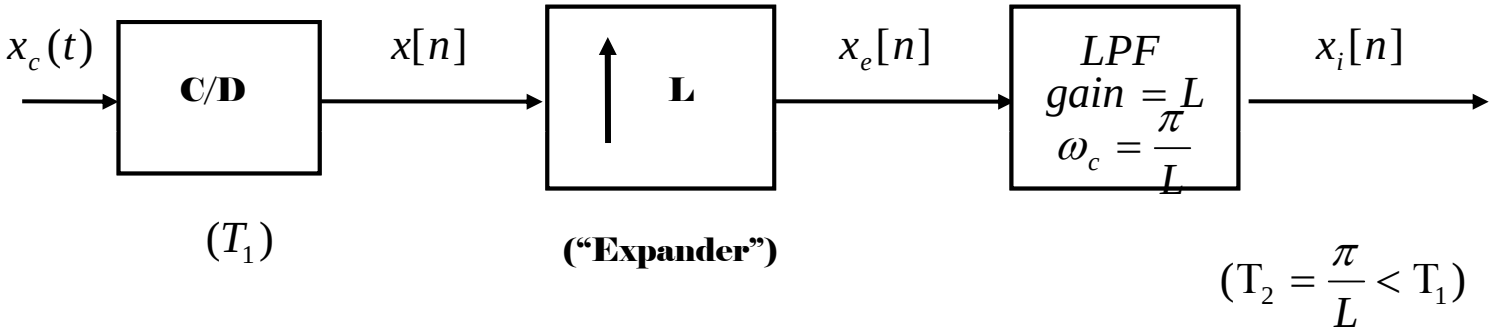

$x_c(t)$
C/D
$x[n]$
L
$x_e[n]$
LPF
$gain = L$
$\omega_c = \frac{\pi}{L}$
$x_i[n]$
(T_1)
("Expander")
$(T_2 = \frac{\pi}{L} < T_1)$

Figure 4.43

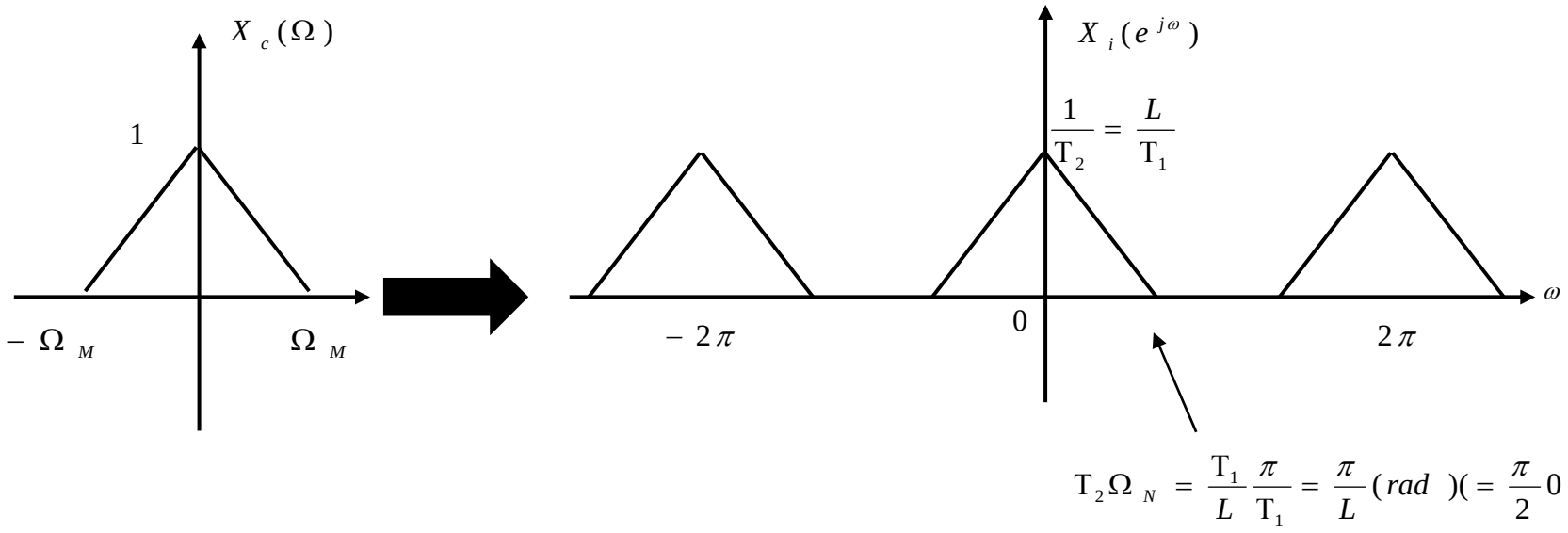

$X_c(\Omega)$
1
$-\Omega_M$
Ω_M
$X_i(e^{j\omega})$
$\frac{1}{T_2} = \frac{L}{T_1}$
-2π
0
2π
ω
$T_2\Omega_N = \frac{T_1}{L}\frac{\pi}{T_1} = \frac{\pi}{L}(rad)(= \frac{\pi}{2}0$

Figure 4.44

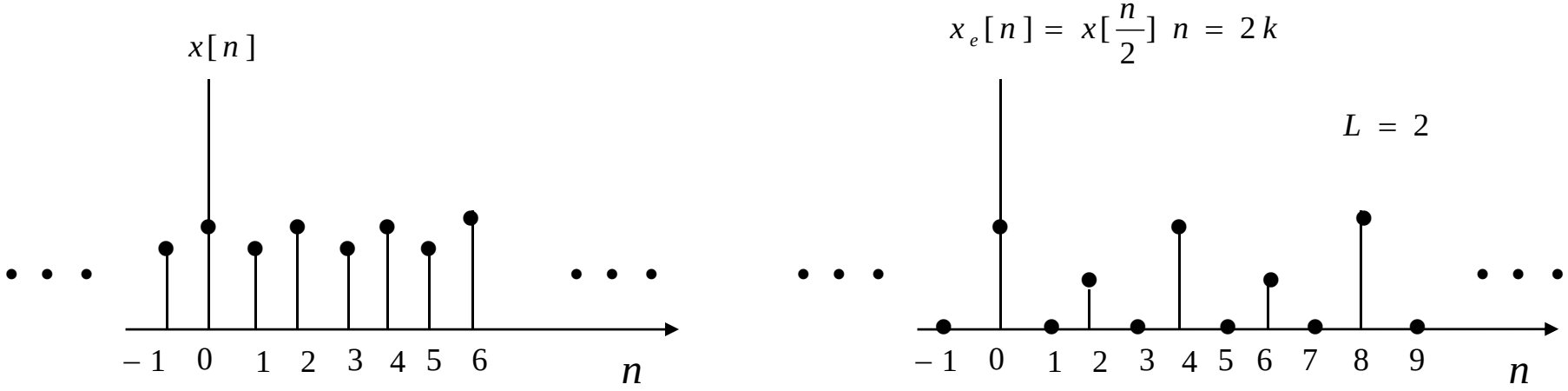


Figure 4.45

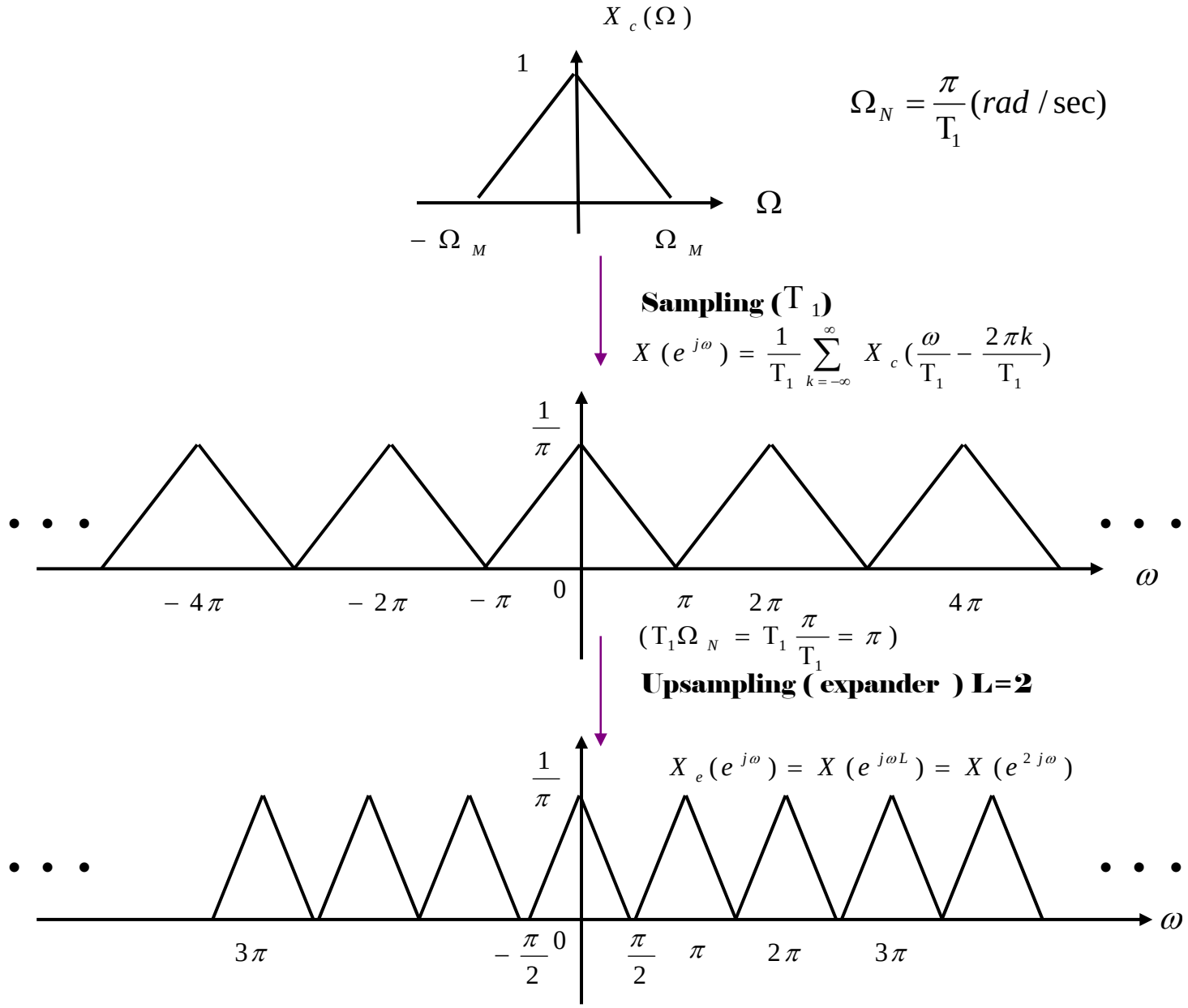

$X_c(\Omega)$
1
Ω
$-\Omega_M$
Ω_M
$\Omega_N = \frac{\pi}{T_1}(rad/\sec)$
Sampling (T_1)
$X(e^{j\omega}) = \frac{1}{T_1}\sum_{k=-\infty}^{\infty} X_c(\frac{\omega}{T_1} - \frac{2\pi k}{T_1})$
$\frac{1}{\pi}$
-4π
-2π
$-\pi$
0
π
2π
4π
ω
$(T_1\Omega_N = T_1\frac{\pi}{T_1} = \pi)$
Upsampling (expander) L=2
$X_e(e^{j\omega}) = X(e^{j\omega L}) = X(e^{2j\omega})$
$\frac{1}{\pi}$
3π
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
2π
3π
ω

Figure 4.46

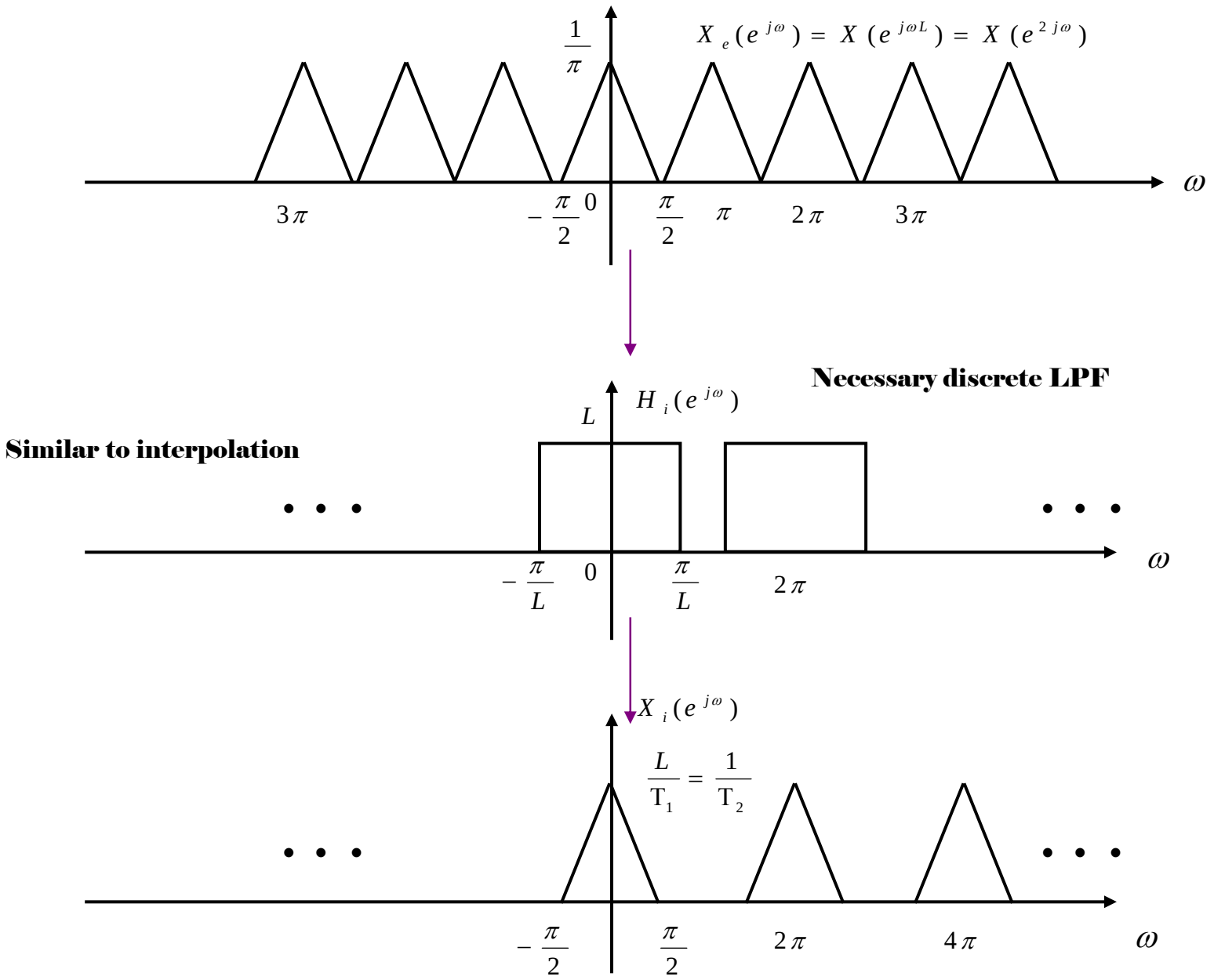

$X_e(e^{j\omega}) = X(e^{j\omega L}) = X(e^{2j\omega})$
$\frac{1}{\pi}$
ω
3π
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
2π
3π
Necessary discrete LPF
Similar to interpolation
$H_i(e^{j\omega})$
L
$-\frac{\pi}{L}$
0
$\frac{\pi}{L}$
2π
$X_i(e^{j\omega})$
$\frac{L}{T_1} = \frac{1}{T_2}$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
2π
4π

Figure 4.46

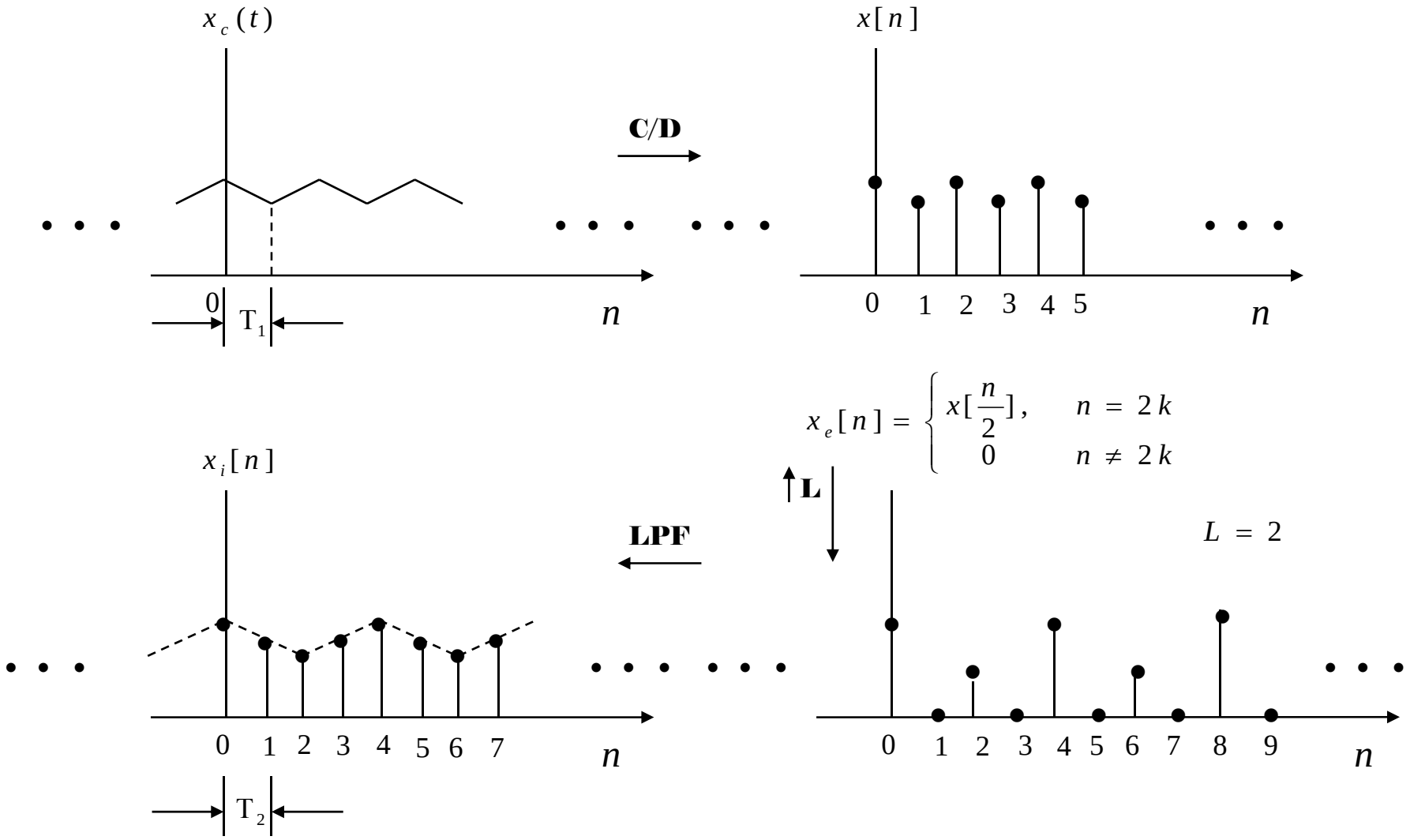

$x_c(t)$
C/D
$x[n]$
0
T_1
n
0 1 2 3 4 5
n
$x_e[n] = \begin{cases} x[\frac{n}{2}], & n = 2k \\ 0 & n \neq 2k \end{cases}$
$x_i[n]$
LPF
L
$L = 2$
0 1 2 3 4 5 6 7
n
T_2
0 1 2 3 4 5 6 7 8 9
n

Figure 4.47

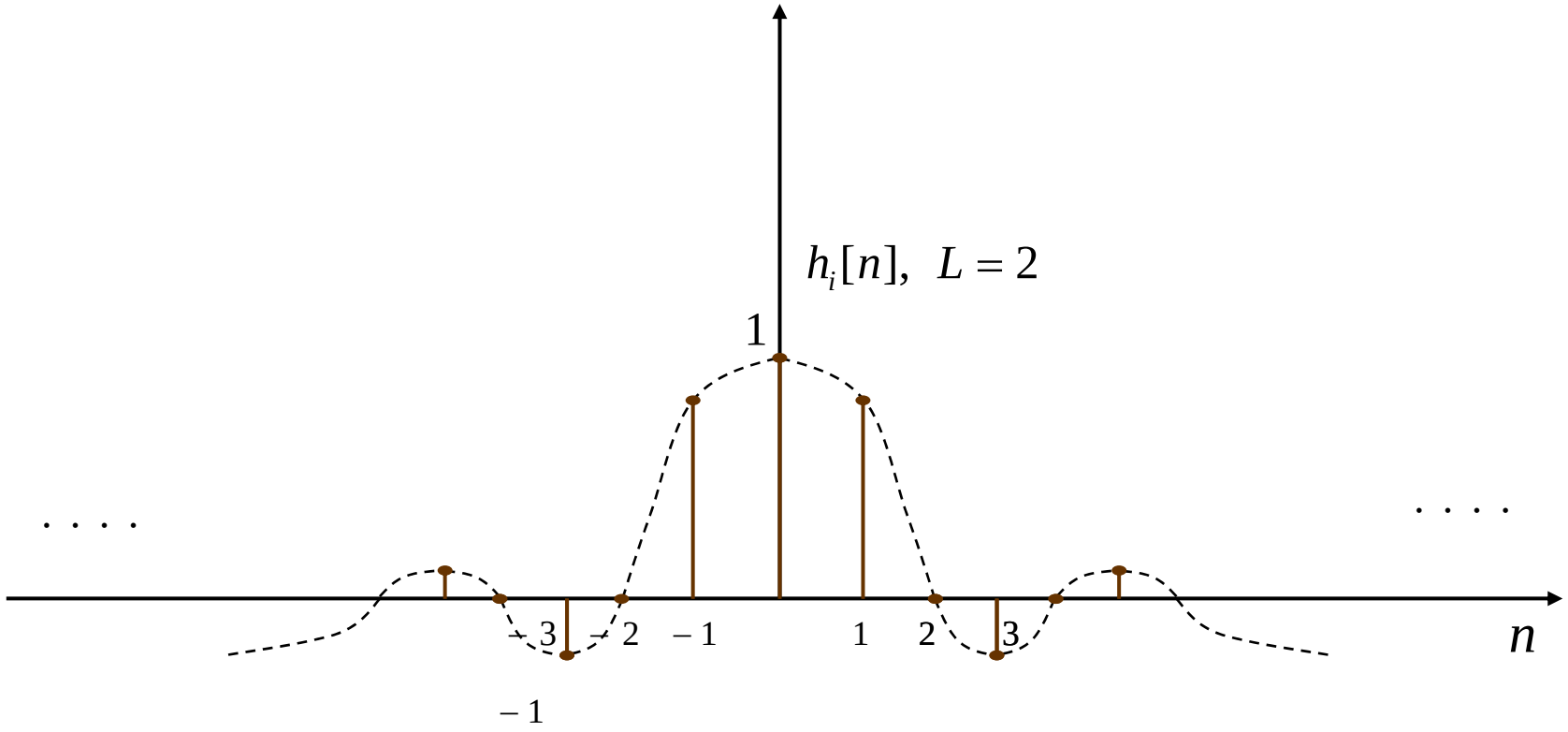


Figure 4.49

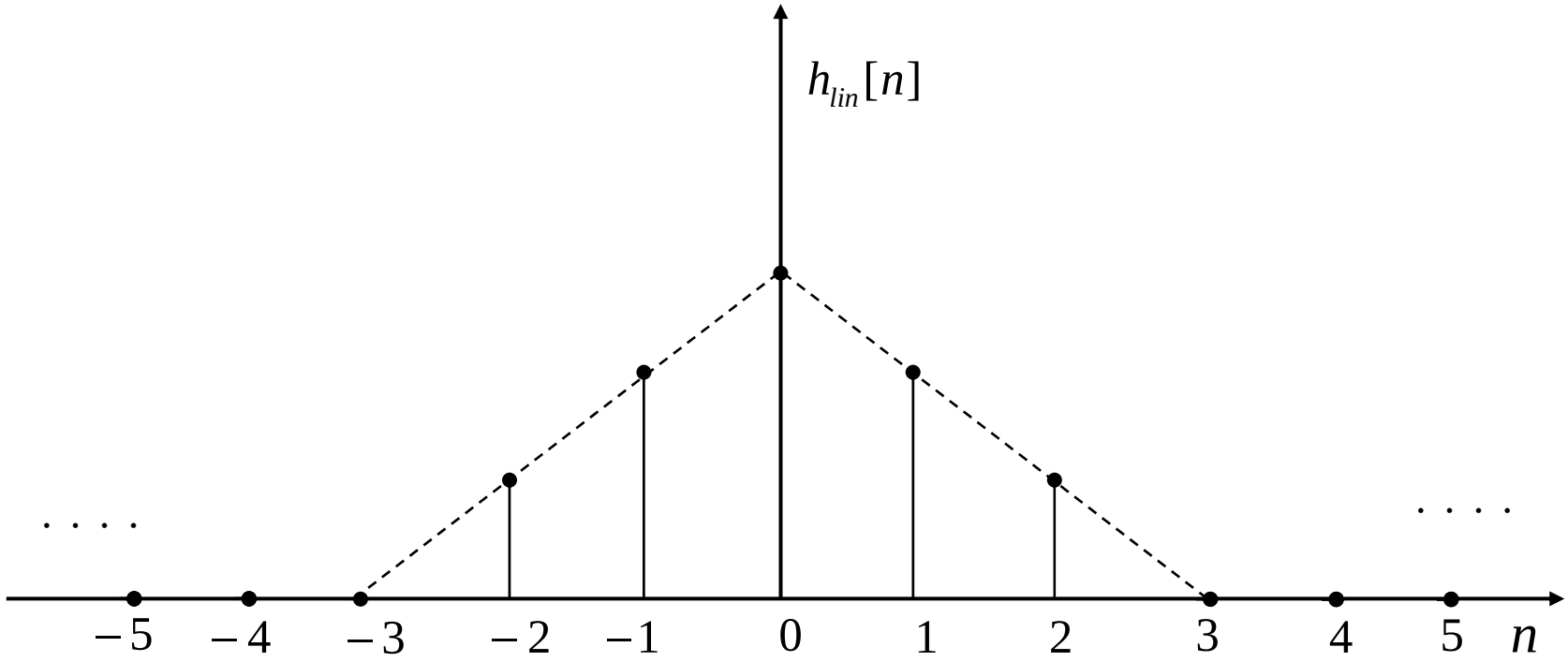


Figure 4.49

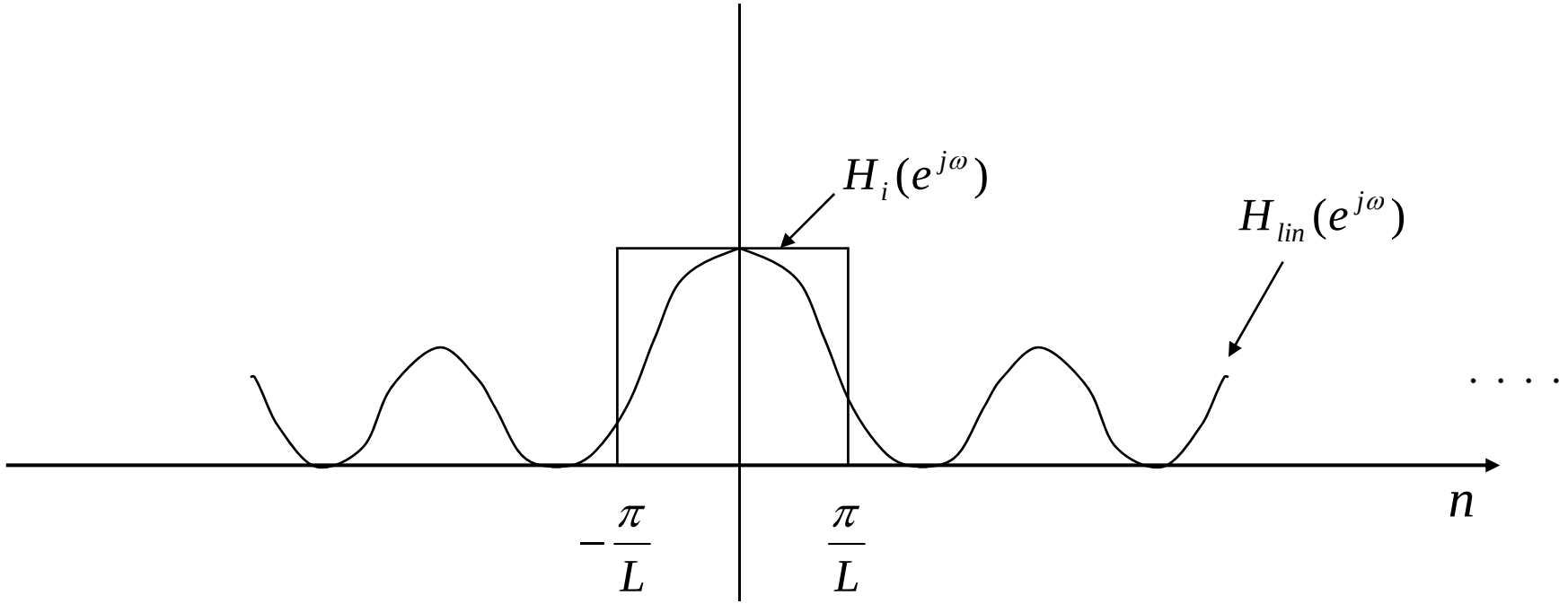


Figure 4.50

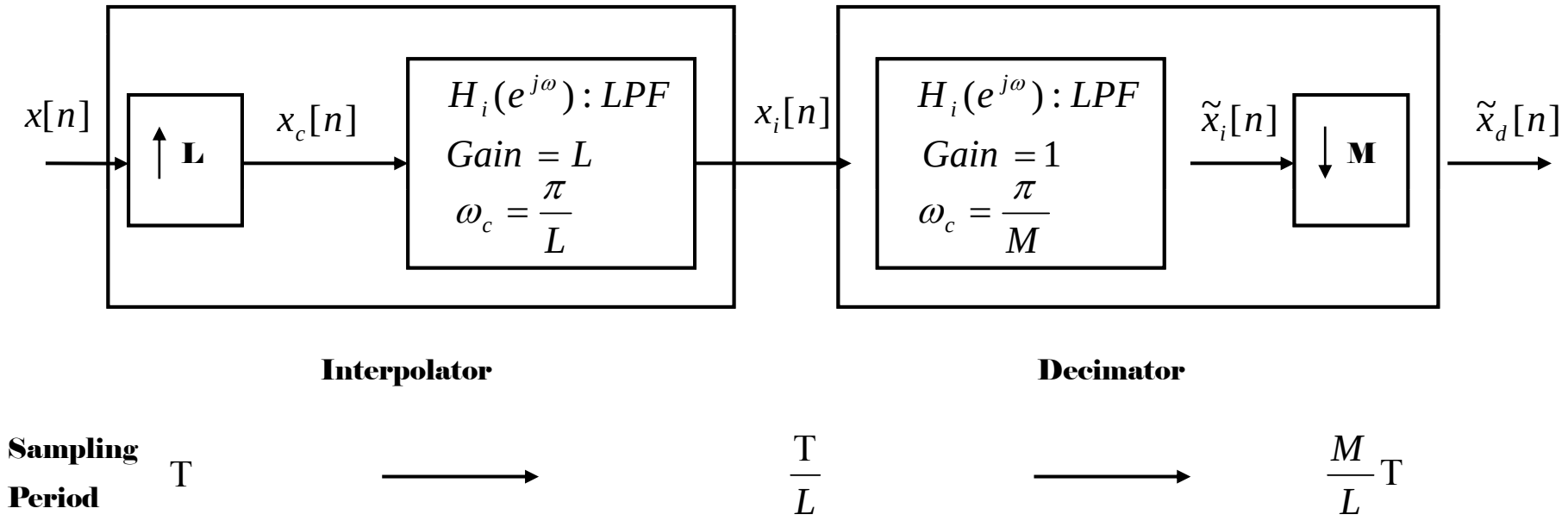

x[n]
L
x_c[n]
H_i(e^{jω}) : LPF
Gain = L
ω_c = π/L
x_i[n]
H_i(e^{jω}) : LPF
Gain = 1
ω_c = π/M
x̃_i[n]
M
x̃_d[n]
Interpolator
Decimator
Sampling Period
T
T/L
M/L T

Figure 4.51

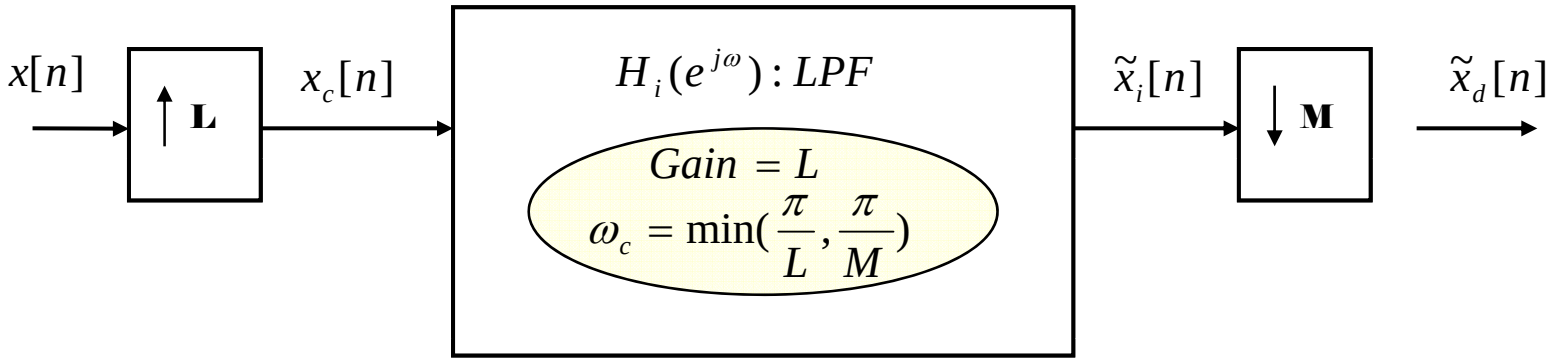


Figure 4.52

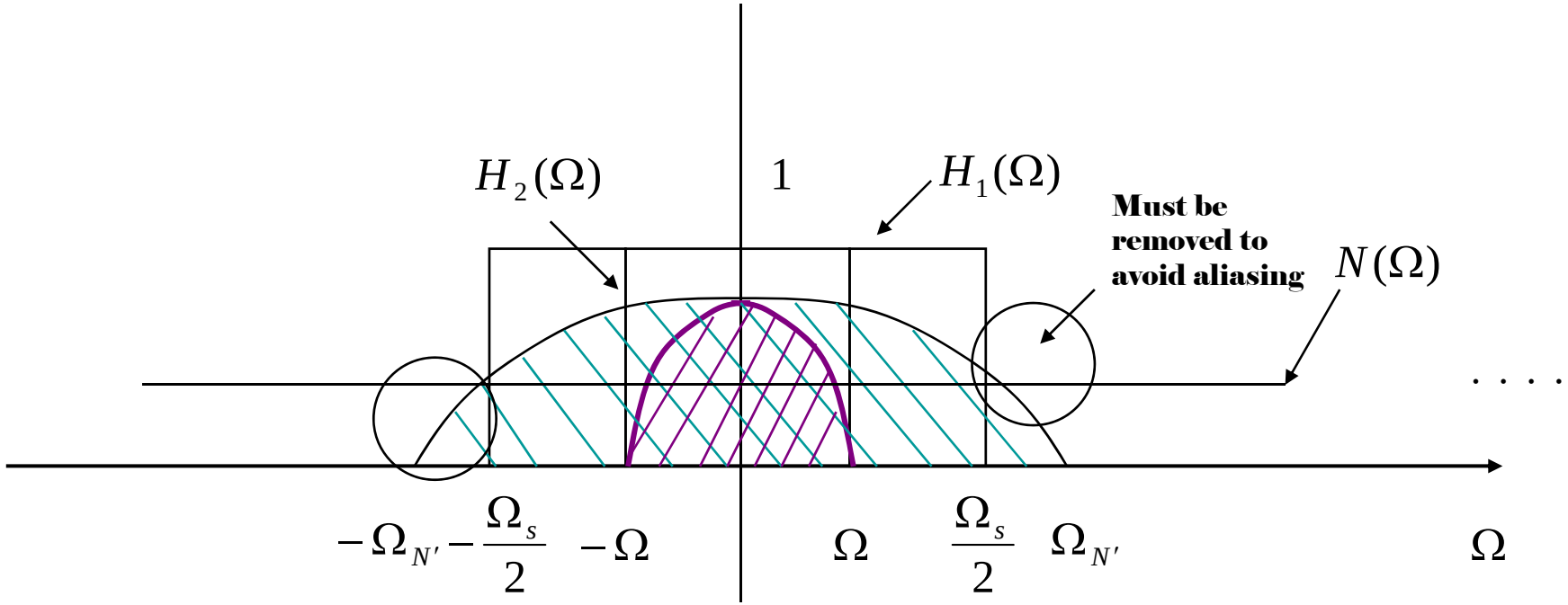

$H_2(\Omega)$
1
$H_1(\Omega)$
Must be removed to avoid aliasing
$N(\Omega)$
. . . .
$-\Omega_{N'}$
$-\frac{\Omega_s}{2}$
$-\Omega$
Ω
$\frac{\Omega_s}{2}$
$\Omega_{N'}$
Ω

Figure 4.53

Figure 4.54

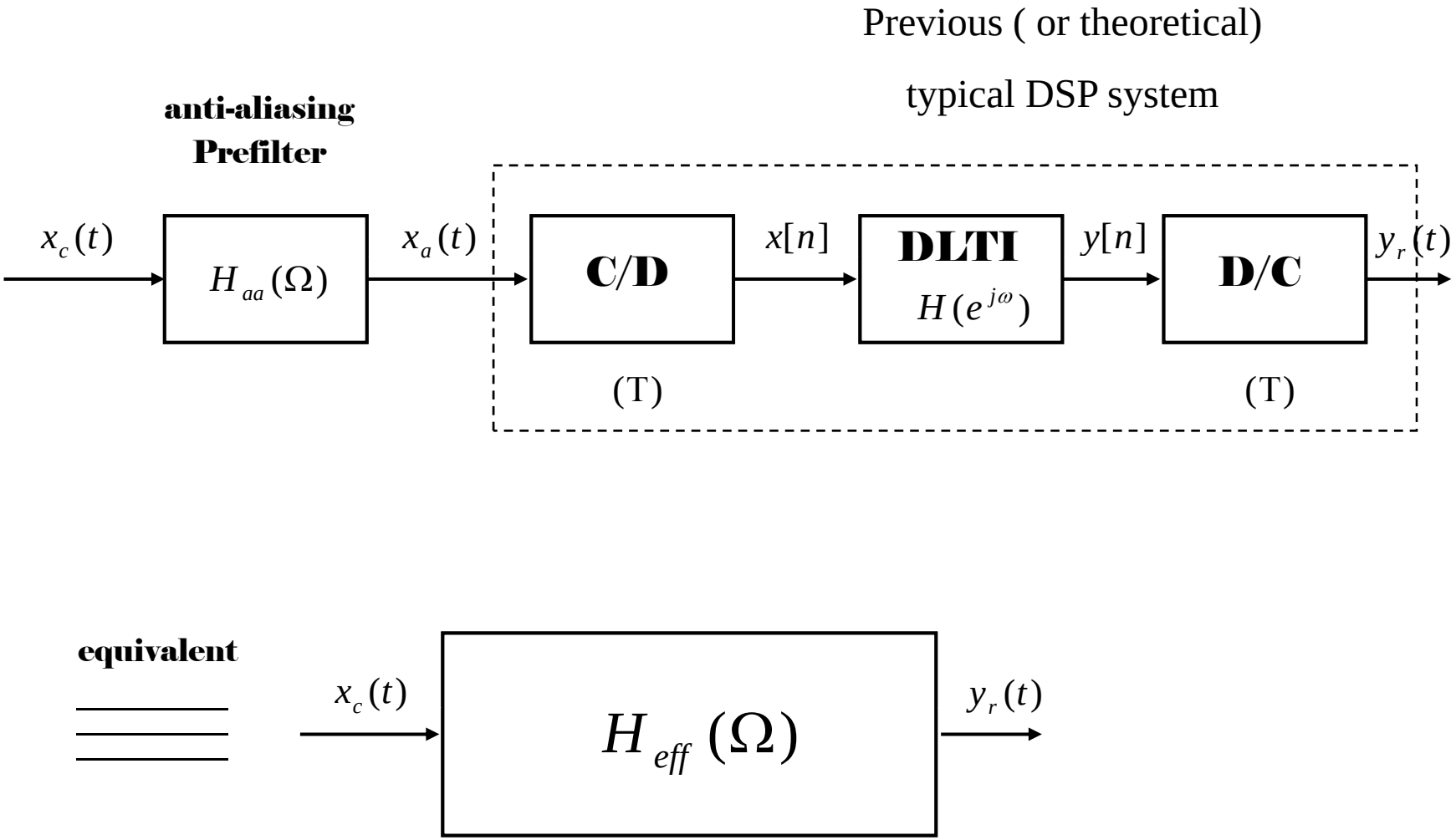

Previous (or theoretical)
typical DSP system
anti-aliasing
Prefilter
$x_c(t)$
$H_{aa}(\Omega)$
$x_a(t)$
C/D
(T)
$x[n]$
DLTI
$H(e^{j\omega})$
$y[n]$
D/C
(T)
$y_r(t)$
equivalent
$x_c(t)$
$H_{eff}(\Omega)$
$y_r(t)$

Figure 4.55

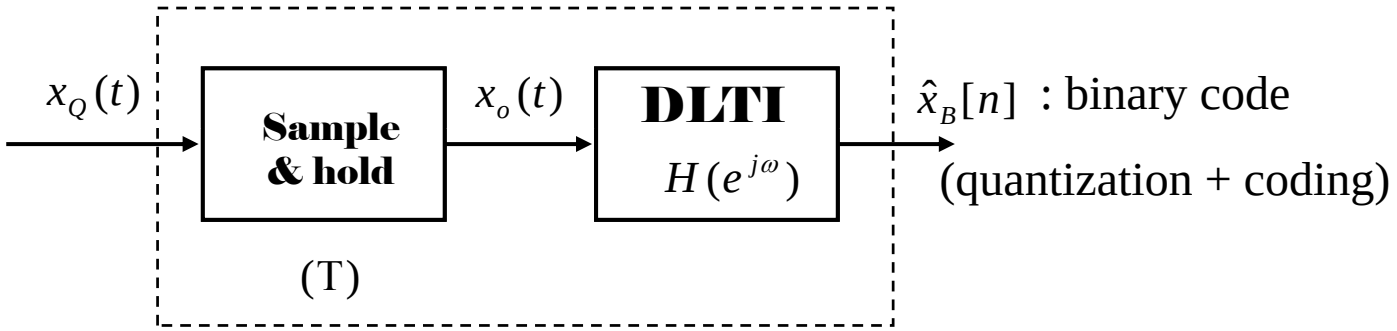

$x_Q(t)$
Sample & hold
$x_o(t)$
DLTI
$H(e^{j\omega})$
$\hat{x}_B[n]$: binary code
(quantization + coding)
(T)

Figure 4.56

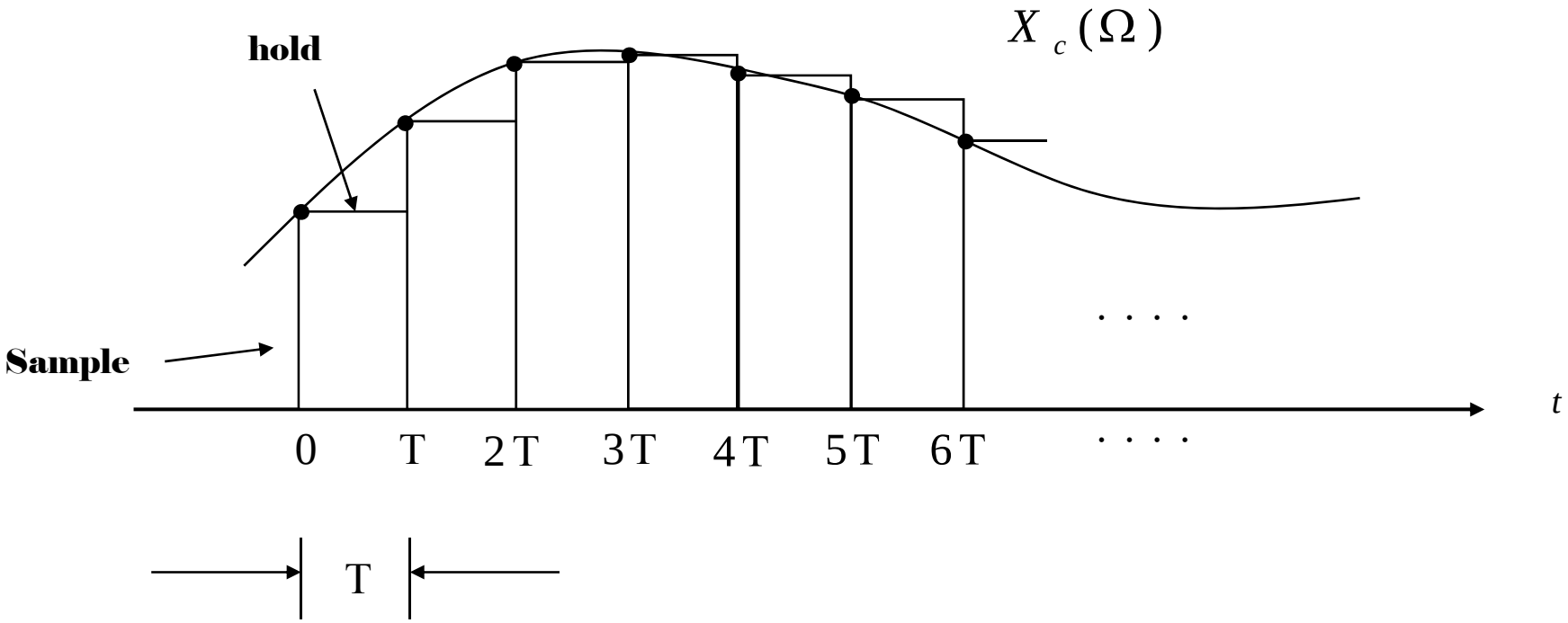


Figure 4.57

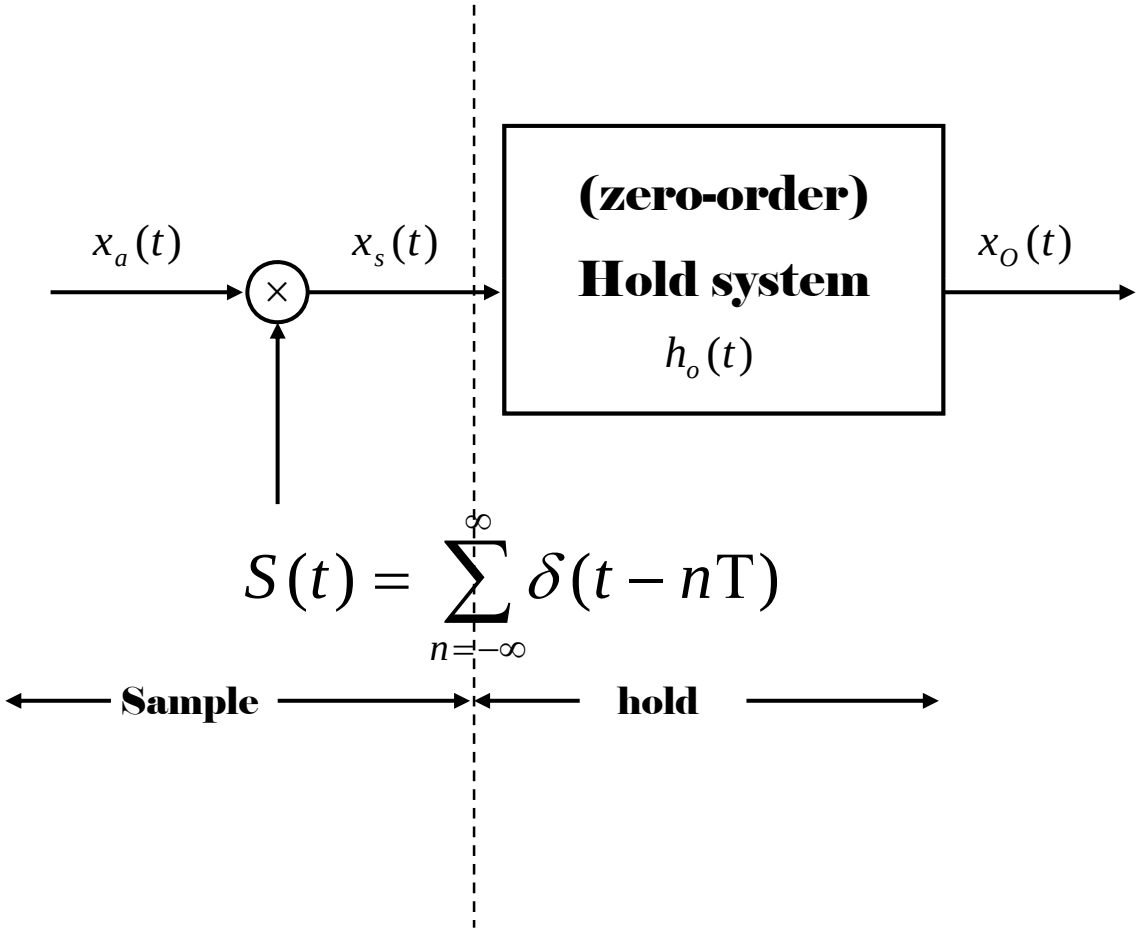

$x_a(t)$
$x_s(t)$
(zero-order)
Hold system
$h_o(t)$
$x_O(t)$
$S(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\mathrm{T})$
Sample
hold

Figure 4.58

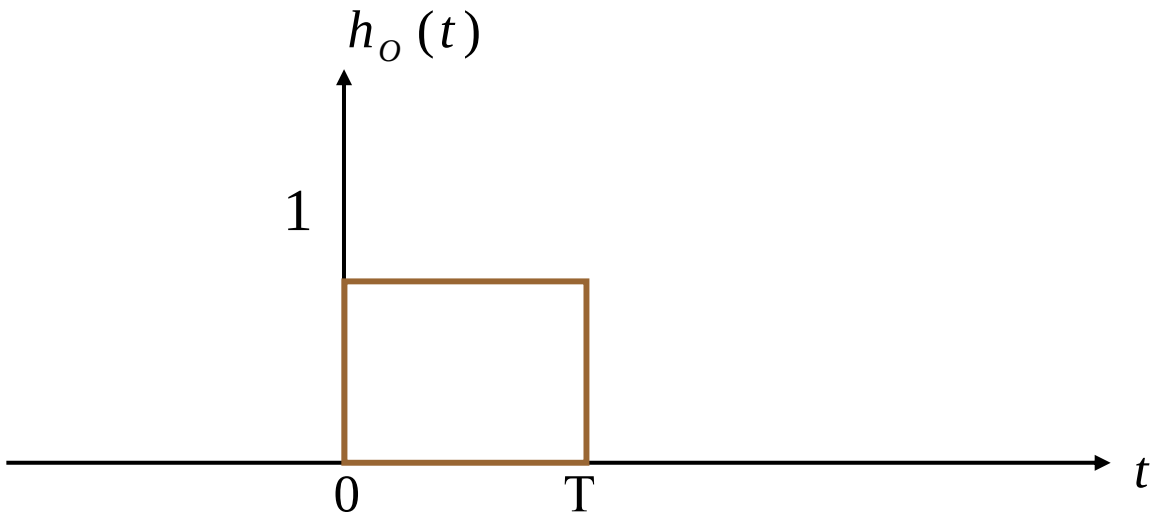


Figure 4.59

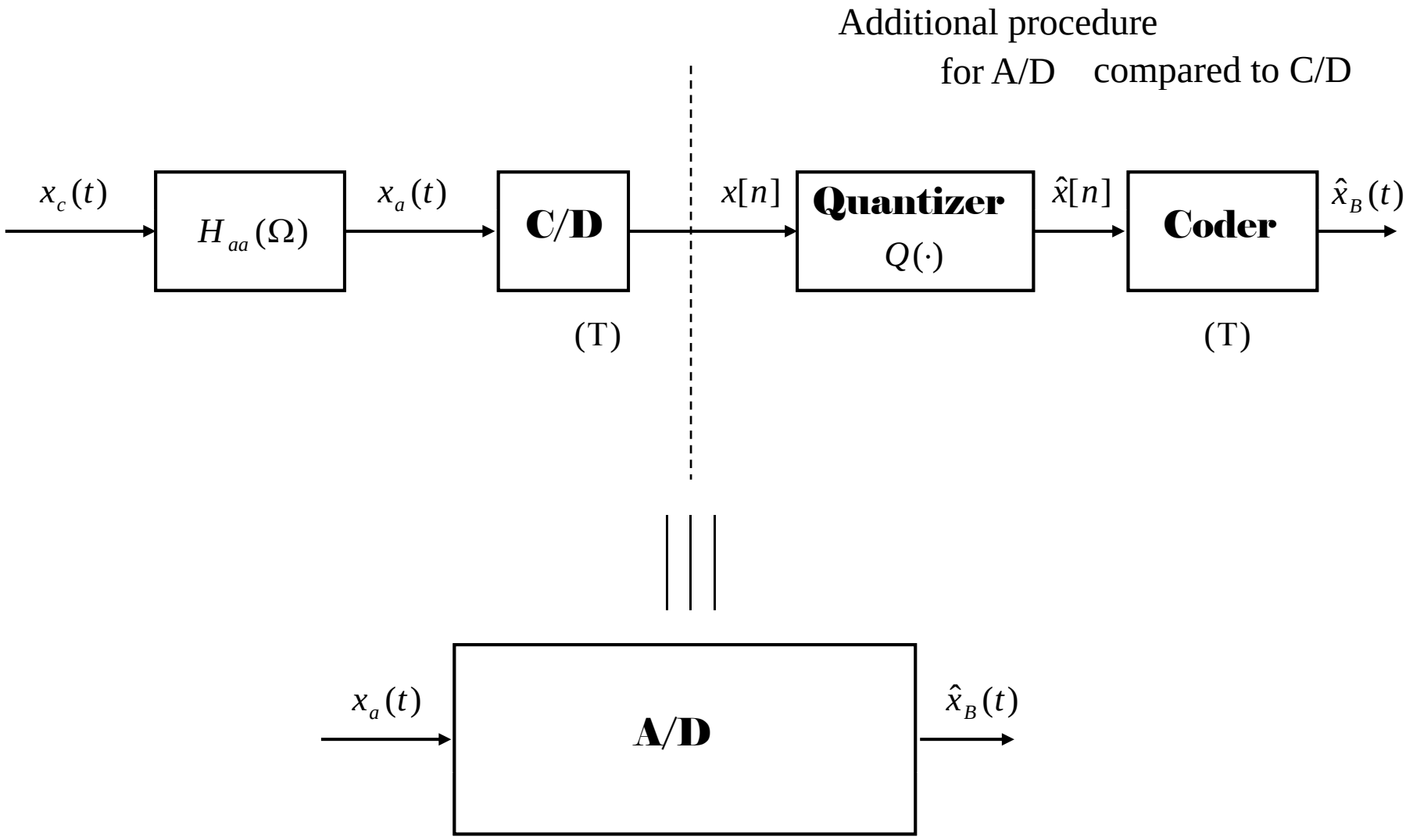

Additional procedure
for A/D compared to C/D
$x_c(t)$
$H_{aa}(\Omega)$
$x_a(t)$
C/D
(T)
$x[n]$
Quantizer
$Q(\cdot)$
$\hat{x}[n]$
Coder
(T)
$\hat{x}_B(t)$
$x_a(t)$
A/D
$\hat{x}_B(t)$

Figure 4.60

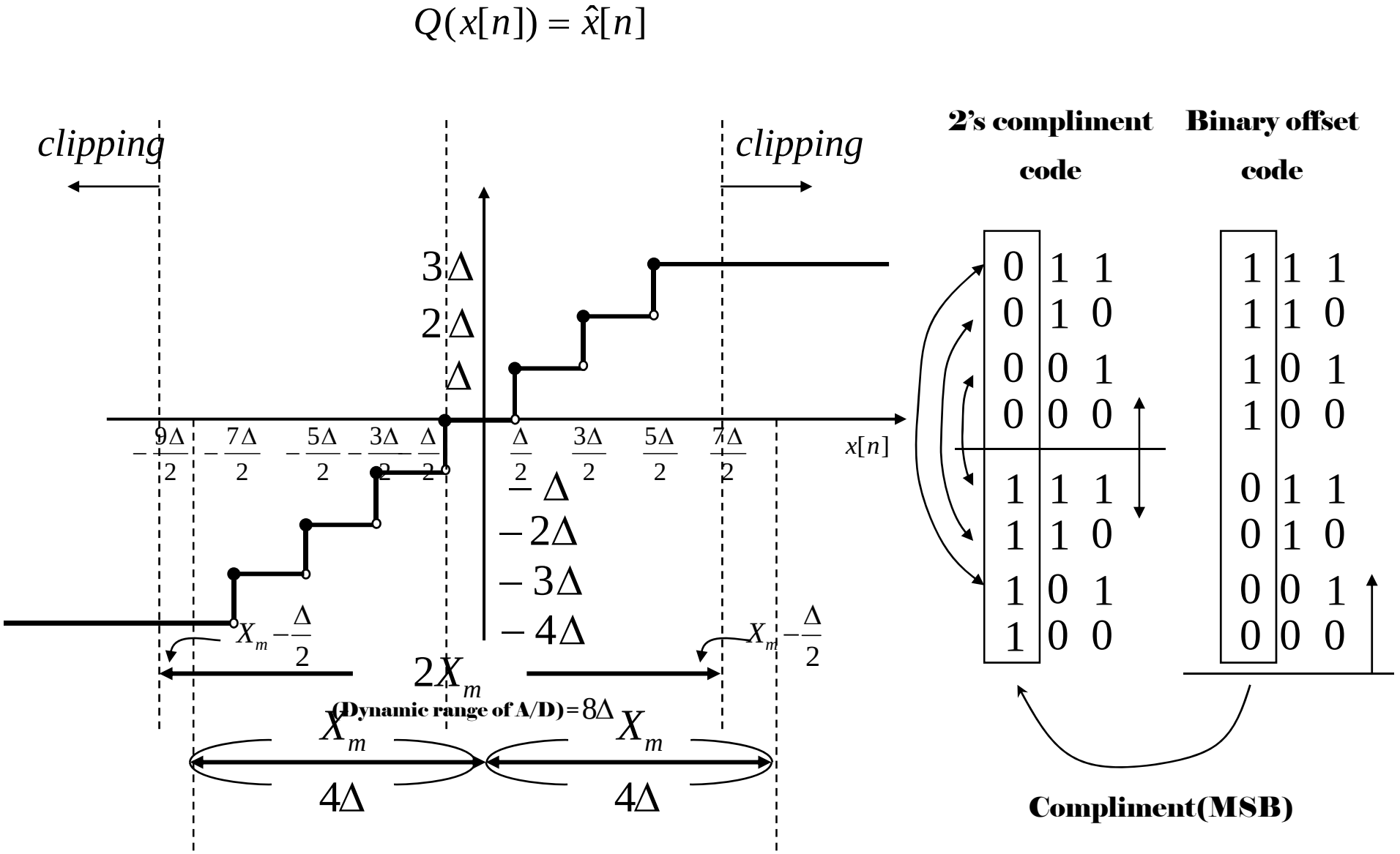

$Q(x[n]) = \hat{x}[n]$
clipping
clipping
2's compliment code
Binary offset code
3Δ
2Δ
Δ
$-\Delta$
-2Δ
-3Δ
-4Δ
$-\frac{9\Delta}{2}$
$-\frac{7\Delta}{2}$
$-\frac{5\Delta}{2}$
$-\frac{3\Delta}{2}$
$-\frac{\Delta}{2}$
$\frac{\Delta}{2}$
$\frac{3\Delta}{2}$
$\frac{5\Delta}{2}$
$\frac{7\Delta}{2}$
$x[n]$
$X_m - \frac{\Delta}{2}$
$X_m - \frac{\Delta}{2}$
$2X_m$
(Dynamic range of A/D) $= 8\Delta$
X_m
X_m
4Δ
4Δ
0 1 1
0 1 0
0 0 1
0 0 0
1 1 1
1 1 0
1 0 1
1 0 0
1 1 1
1 1 0
1 0 1
1 0 0
0 1 1
0 1 0
0 0 1
0 0 0
Compliment(MSB)

Figure 4.61

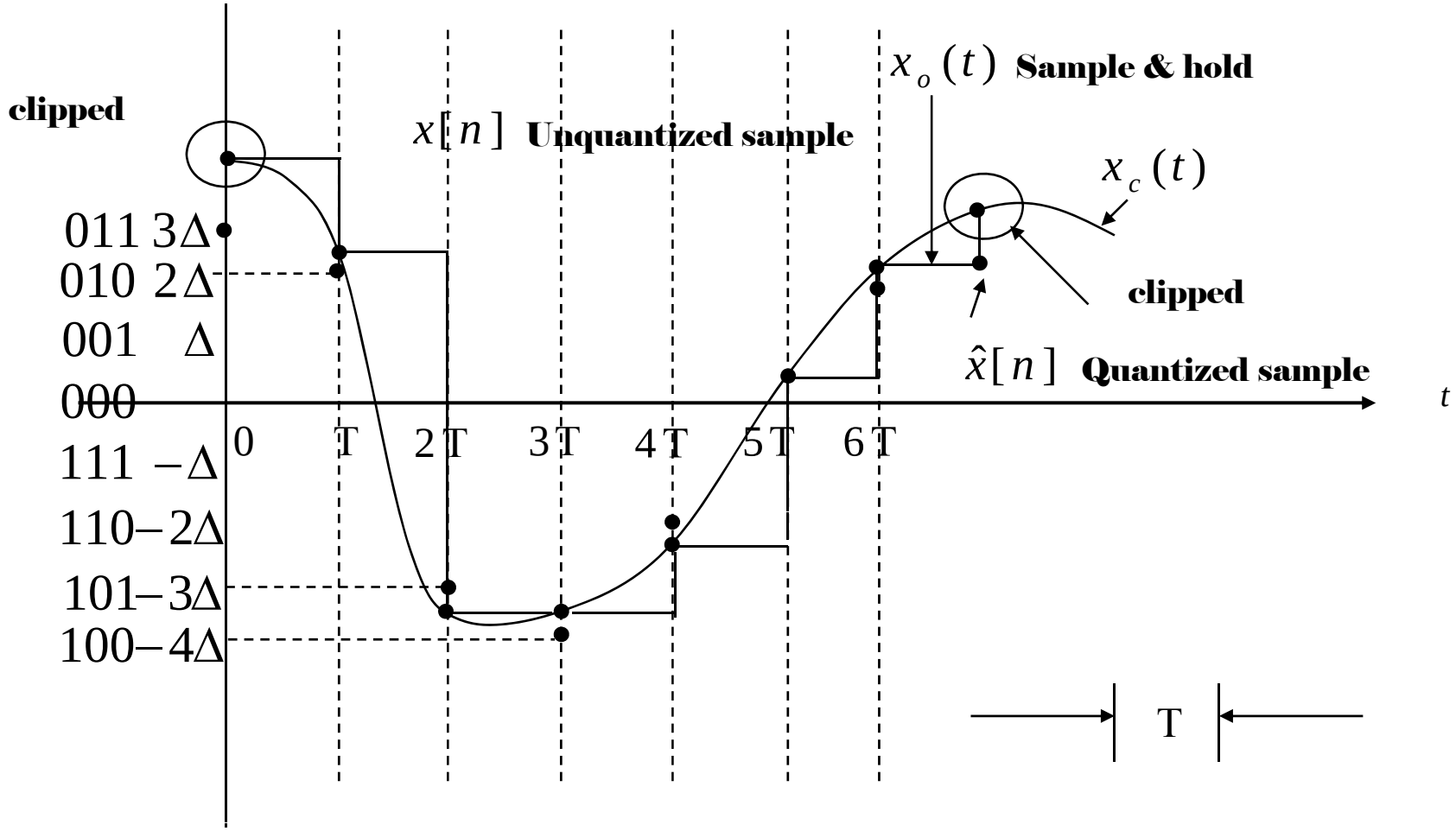

clipped
$x_o(t)$ Sample & hold
$x[n]$ Unquantized sample
$x_c(t)$
011 3Δ
010 2Δ
001 Δ
000
111 −Δ
110 −2Δ
101 −3Δ
100 −4Δ
clipped
$\hat{x}[n]$ Quantized sample
t
0
T
2T
3T
4T
5T
6T
T

Figure 4.62

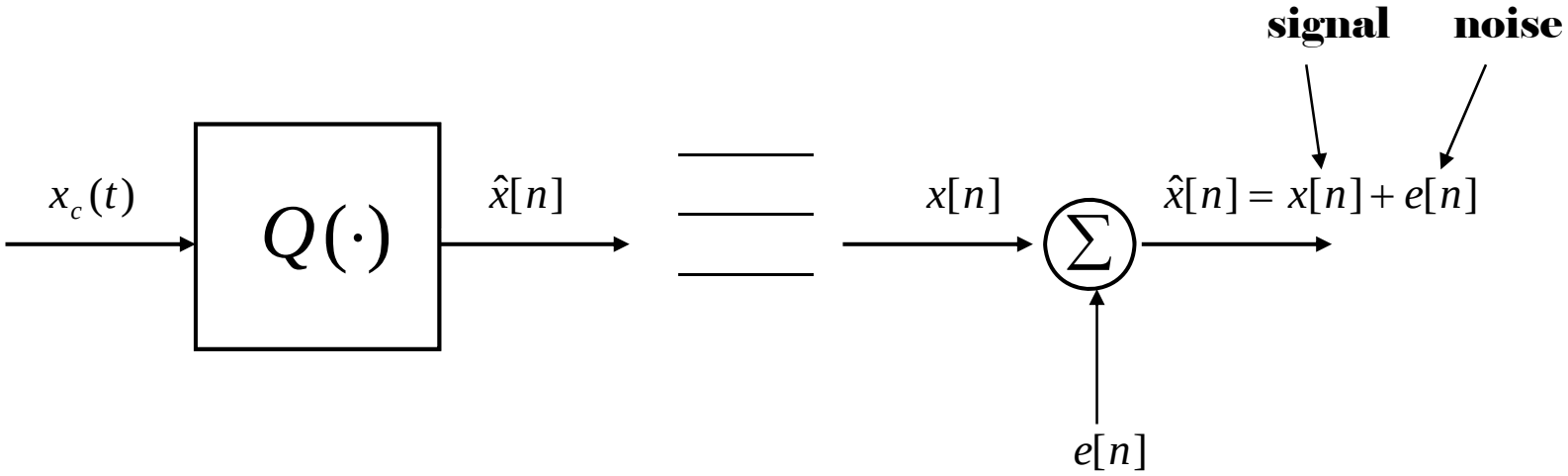
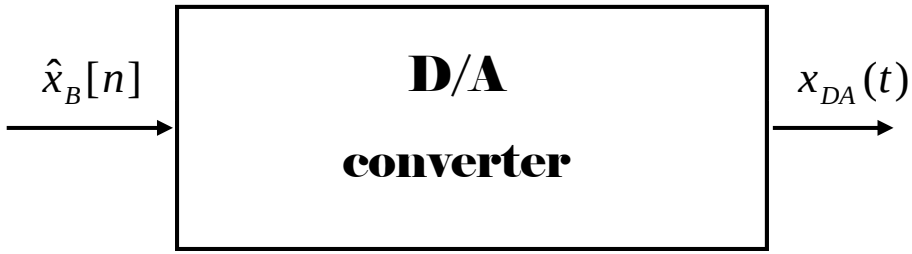


Figure 4.63



Replace in place of the ideal LPF $H_r(\Omega)$

(Reconstruction filter)

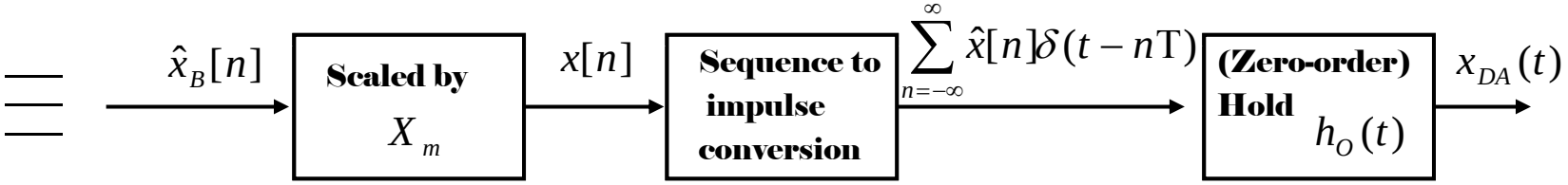


Figure 4.64

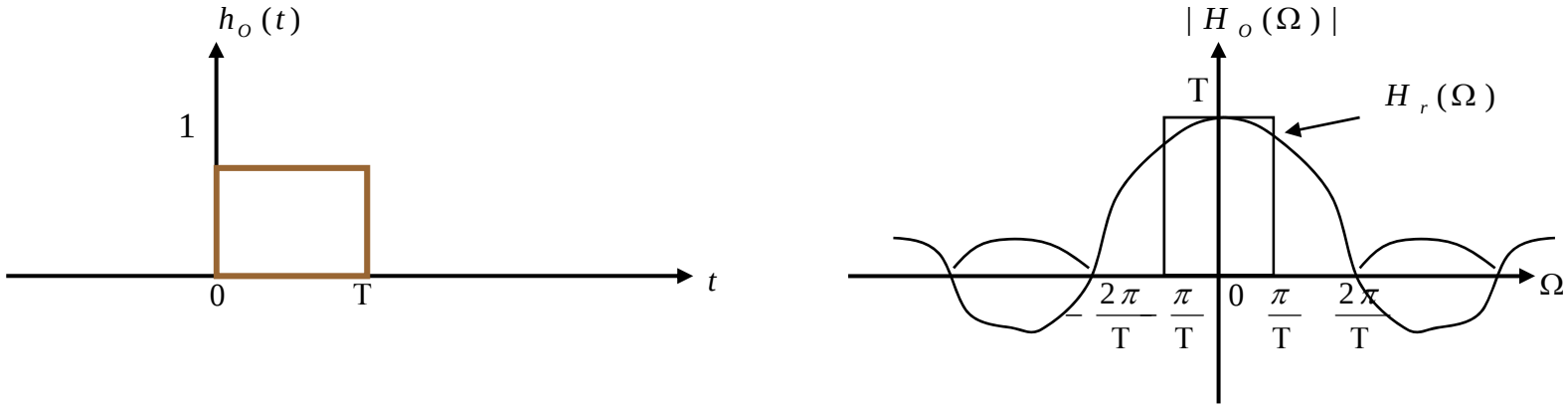

$h_O(t)$
1
0
T
t
$|H_O(\Omega)|$
T
$H_r(\Omega)$
$-\frac{2\pi}{T}$
$-\frac{\pi}{T}$
0
$\frac{\pi}{T}$
$\frac{2\pi}{T}$
Ω

Figure 4.65

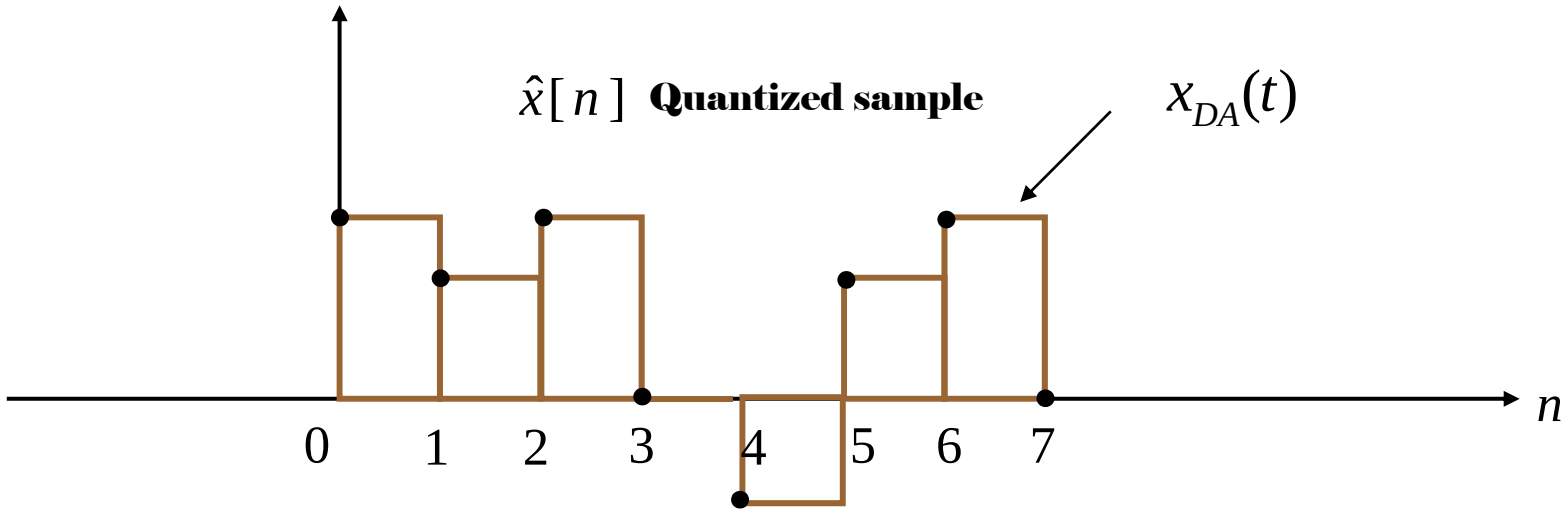
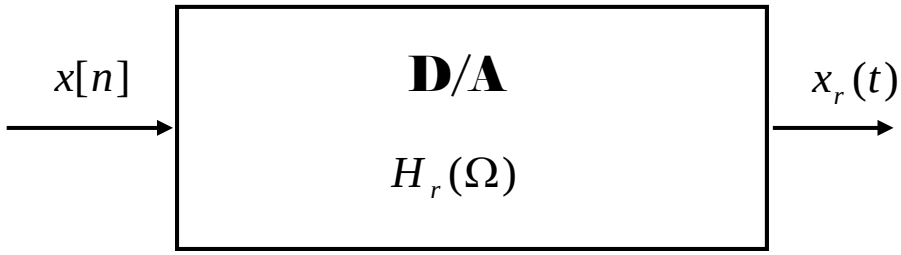


Figure 4.66



Compare w/ $H_o(\Omega)$ above!

(Reconstruction filter)

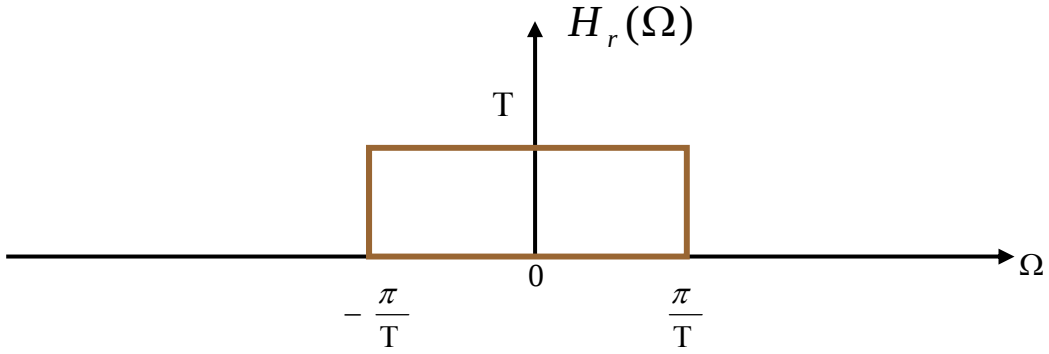


Figure 4.67

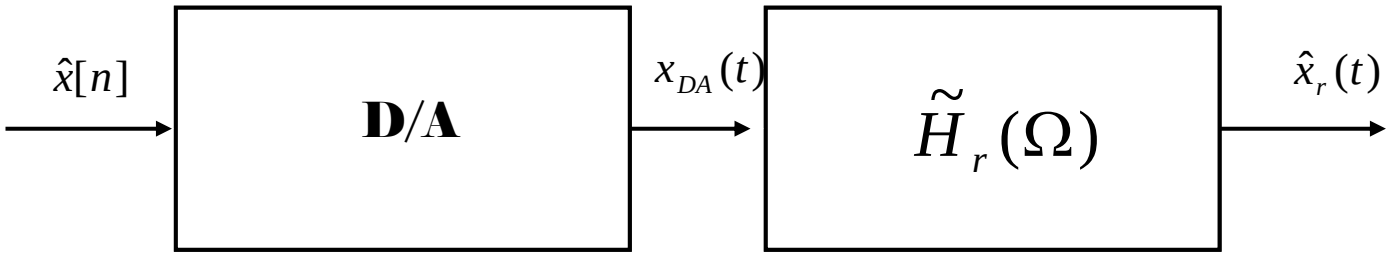


Figure 4.68

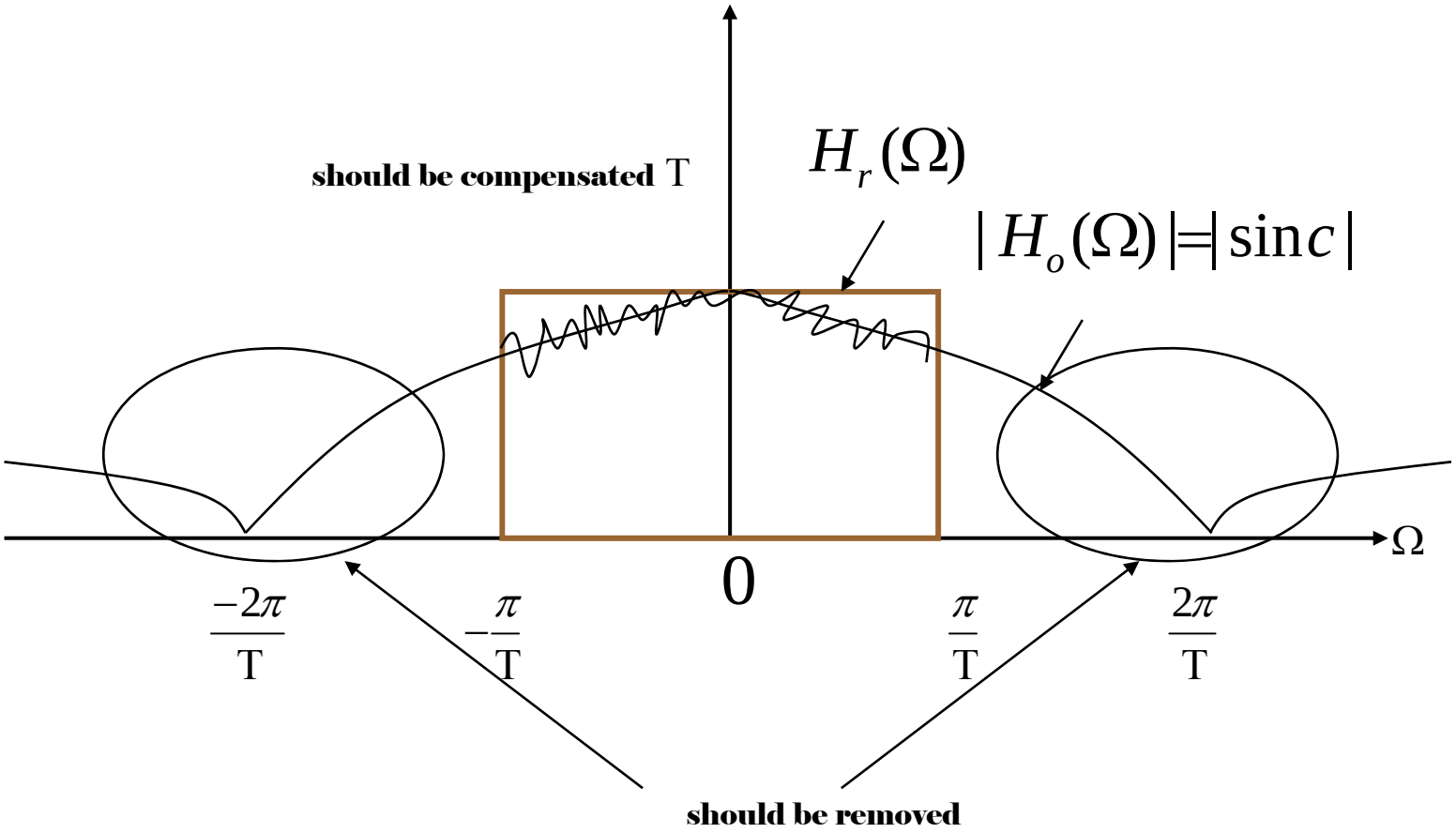

should be compensated T
$H_r(\Omega)$
$|H_o(\Omega)|=|\mathrm{sinc}|$
Ω
0
$\frac{-2\pi}{T}$
$-\frac{\pi}{T}$
$\frac{\pi}{T}$
$\frac{2\pi}{T}$
should be removed

Figure 4.69

$$|\tilde{H}_r(\Omega)| \approx \tilde{H}_r(\Omega)$$

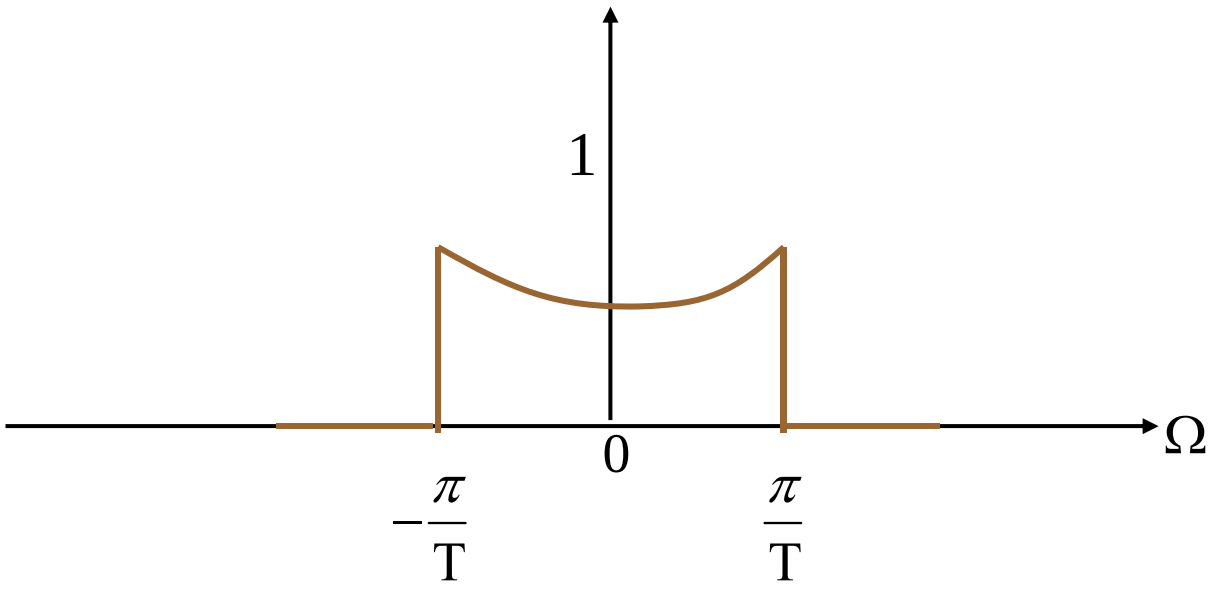


Figure 4.70

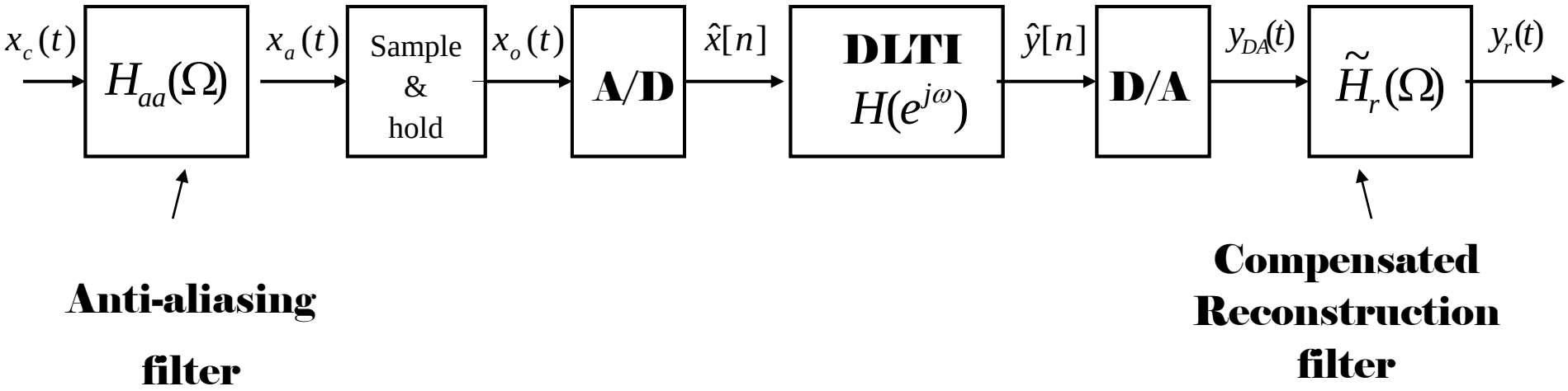

$x_c(t)$
$H_{aa}(\Omega)$
$x_a(t)$
Sample & hold
$x_o(t)$
A/D
$\hat{x}[n]$
DLTI
$H(e^{j\omega})$
$\hat{y}[n]$
D/A
$y_{DA}(t)$
$\tilde{H}_r(\Omega)$
$y_r(t)$
Anti-aliasing filter
Compensated Reconstruction filter

Figure 4.71

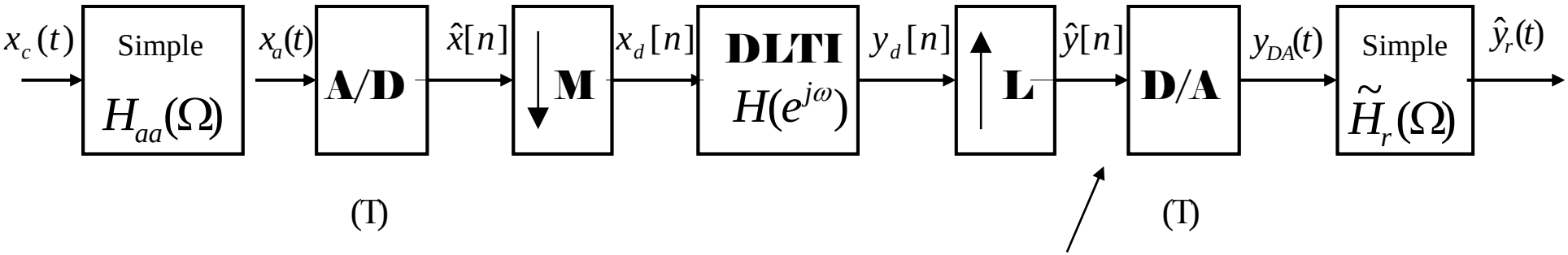

$x_c(t)$
Simple
$H_{aa}(\Omega)$
$x_a(t)$
A/D
(T)
$\hat{x}[n]$
M
$x_d[n]$
DLTI
$H(e^{j\omega})$
$y_d[n]$
L
$\hat{y}[n]$
D/A
(T)
$y_{DA}(t)$
Simple
$\tilde{H}_r(\Omega)$
$\hat{y}_r(t)$
Compensated Reconstruction Filter(conti)
Incorporated into interpolation filter (discrete)

Figure 4.72

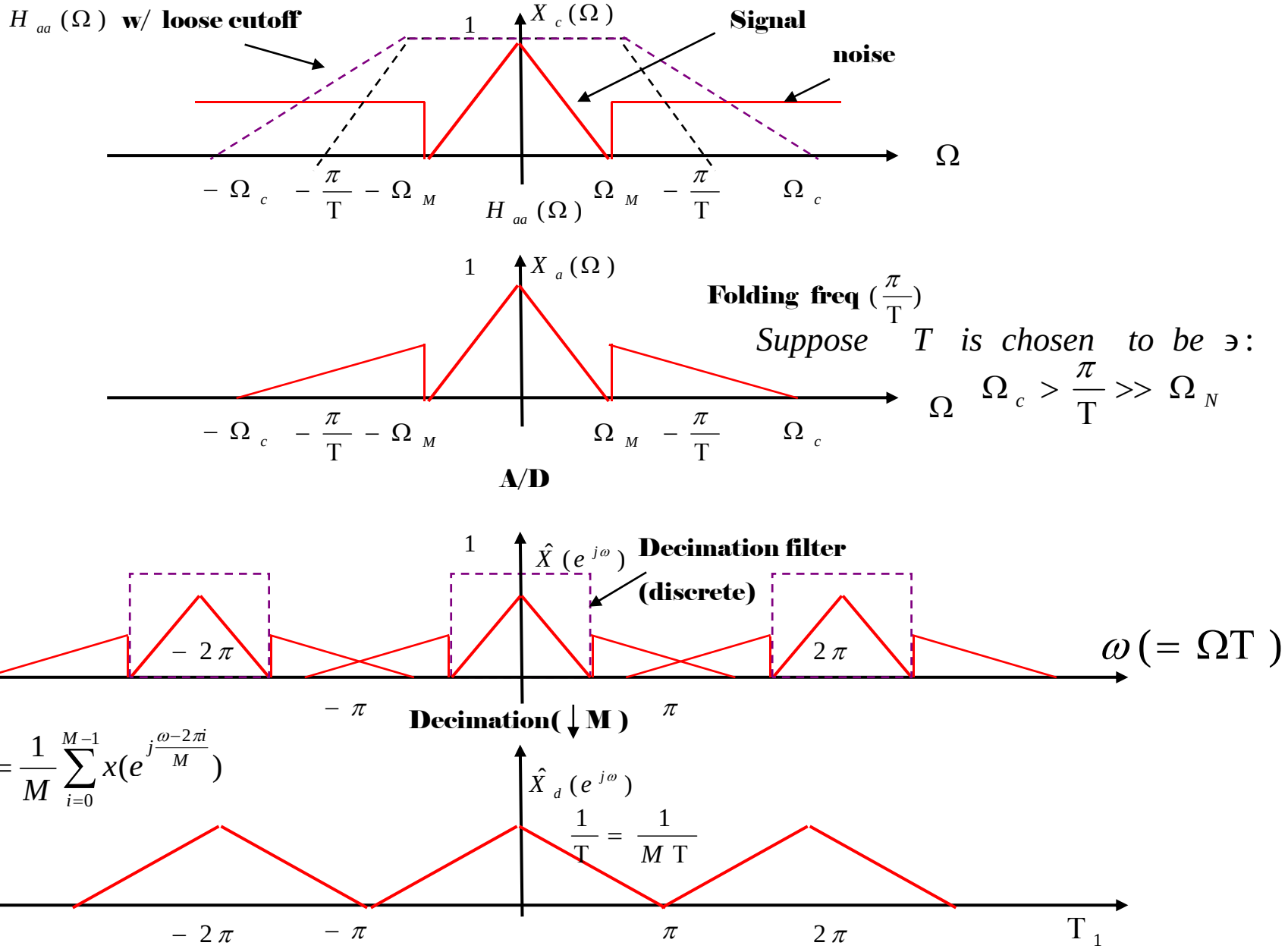

$H_{aa}(\Omega)$ w/ loose cutoff
1
$X_c(\Omega)$
Signal
noise
Ω
$-\Omega_c$ $-\frac{\pi}{T}$ $-\Omega_M$
$H_{aa}(\Omega)$
Ω_M $-\frac{\pi}{T}$ Ω_c
1
$X_a(\Omega)$
Folding freq $(\frac{\pi}{T})$
Suppose T is chosen to be ∋:
$\Omega_c > \frac{\pi}{T} >> \Omega_N$
Ω
$-\Omega_c$ $-\frac{\pi}{T}$ $-\Omega_M$
Ω_M $-\frac{\pi}{T}$ Ω_c
A/D
1
$\hat{X}(e^{j\omega})$
Decimation filter
(discrete)
-2π
2π
$\omega(=\Omega T)$
$-\pi$
π
Decimation($\downarrow$M)
$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} x(e^{j\frac{\omega-2\pi i}{M}})$
$\hat{X}_d(e^{j\omega})$
$\frac{1}{T} = \frac{1}{MT}$
-2π
$-\pi$
π
2π
T_1

Figure 4.73

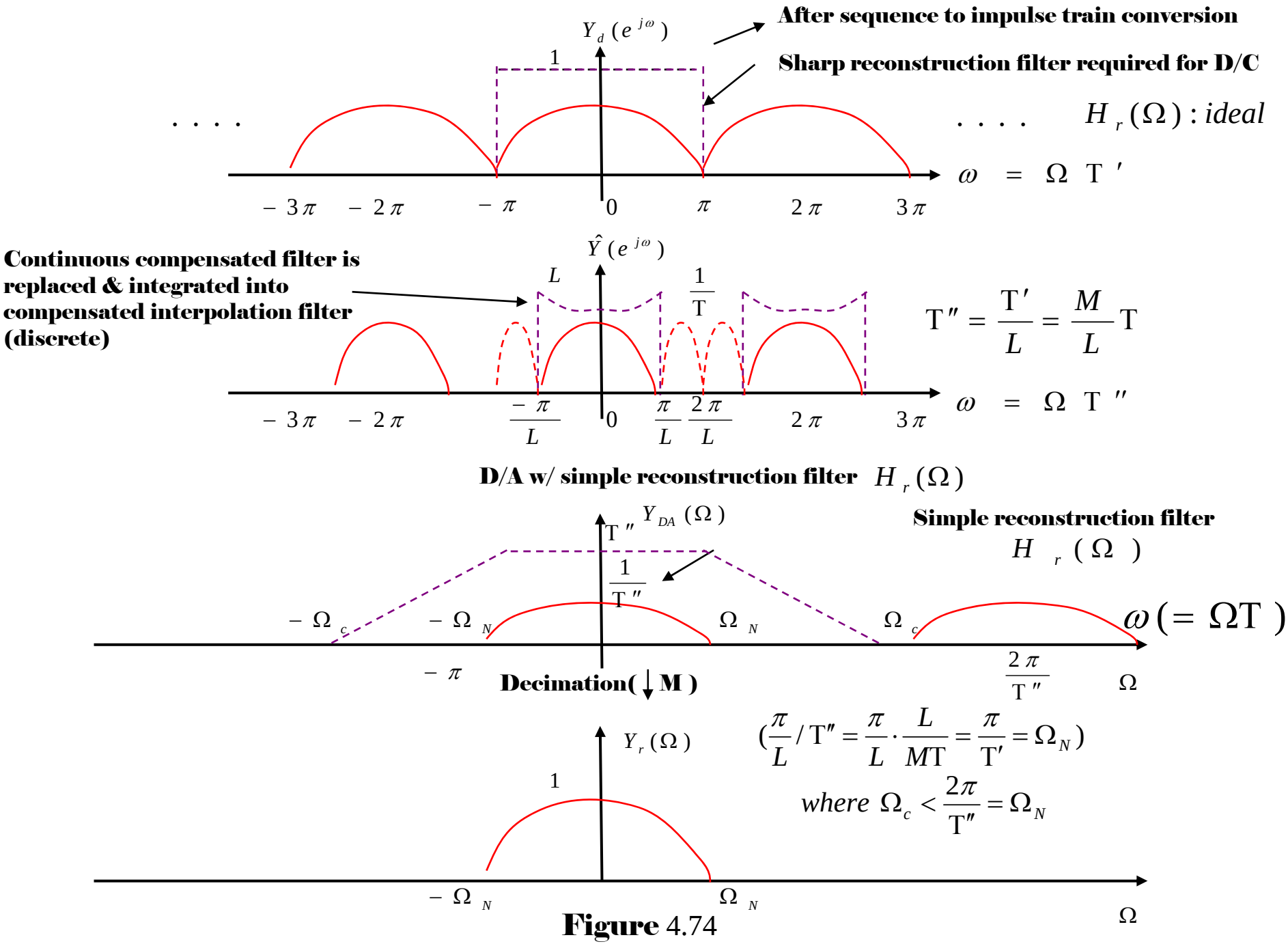

After sequence to impulse train conversion
Sharp reconstruction filter required for D/C
$Y_d(e^{j\omega})$
1
$H_r(\Omega)$: ideal
$\omega = \Omega T'$
-3π -2π $-\pi$ 0 π 2π 3π
Continuous compensated filter is replaced & integrated into compensated interpolation filter (discrete)
$\hat{Y}(e^{j\omega})$
L
$\frac{1}{T}$
$T'' = \frac{T'}{L} = \frac{M}{L}T$
$\omega = \Omega T''$
$-\frac{\pi}{L}$ $\frac{\pi}{L}$ $\frac{2\pi}{L}$
D/A w/ simple reconstruction filter $H_r(\Omega)$
$Y_{DA}(\Omega)$
T''
$\frac{1}{T''}$
Simple reconstruction filter
$H_r(\Omega)$
$-\Omega_c$ $-\Omega_N$ Ω_N Ω_c
$\omega(=\Omega T)$
$-\pi$
$\frac{2\pi}{T''}$
Decimation($\downarrow M$)
$Y_r(\Omega)$
$(\frac{\pi}{L}/T'' = \frac{\pi}{L}\cdot\frac{L}{MT} = \frac{\pi}{T'} = \Omega_N)$
where $\Omega_c < \frac{2\pi}{T''} = \Omega_N$
$-\Omega_N$ Ω_N Ω

Figure 4.74