Contents

Chapter 1

INTRODUCTION

1.1 Introduction

Purposes of class (objective) :

(1) Underlying ideas of probability and statistics

(2) Engineering applications of probability and statistics

Definition 1.1 Probability:
Deals with averages of mass, unpredictable phenomena

Definition 1.2 Statistics:
Concerned with collection and representation of engineering data so that practical conclusion can be drawn

Classification of natural phenomena: [1]

(A) Deterministic phenomenon

(B) Random phenomenon

[1] Most of the phenomena(especially signals) you have been dealing with are deterministic, although it can be argued that nothing in nature is truly deterministic. You should not have to think too hard to realize that not every experiment, either naturally occurring or otherwise, can be treated as deterministic. The birth of male and female human beings, for example, is a random phenomenon where, on the average, the ratio of male to female births appears to be about 51% to 49% respectively.

Mathematical Model: significance and procedure

$$\boxed{\text{Identification of model}}$$

$$\Downarrow$$

$$\boxed{\text{Solution for certain quantities of interest (mathematics)}}$$

$$\Downarrow$$

$$\boxed{\text{Verification of model (physical experiment)}}$$

$$\Downarrow$$

$$\boxed{\text{Modification of model (based on experimental results)}}$$

Usage of mathematical model:

(1) Applying to similar other situations & predicting the outcome (*Analysis*)

(2) Suggesting alternative solution for a given problem(*Design*)

Why probabilistic approaches to engineering problems?

1. Unable to model perfectly some aspects of actual situation
2. Unable to make perfect measurements of physical quantities
3. Some phenomena in nature are “random” inherently

$\Longrightarrow$ *Indispensable tool in engineering problems*

1.2 Two Examples

- Difference b/w deterministic and probabilistic models
- Some ways of summarizing random data

Example 1.1

Measurements of voltage levels:

(1) Internal rms noise of meter: 1 μV

(2) Voltages to be measured: (a) 1 V, (b) 0.02 mV, (c) 5 μV

(3) Estimate: Average of 50 (V_i, $i = 1, 2, \cdots, 50$) measurements (sample mean)

$$V_{sm} = \frac{1}{50} \sum_{i=1}^{50} V_i \tag{1.1}$$

(4) Results(V_{sm}): (a) 1 V, (b) 0.0197 mV, (c) 4.97 μV

Figure 1.1: Measurement of 3 noisy voltages: (a) $1V$; (b) $20\mu V$; (c) $5\mu V$. The rms noise level is $1\mu V$.

Example 1.2

Examination grades in a class of students:

(1) Graded points(25 students):
97, 93, 92, 87, 85, 84, 83, 80, 80, 77, 76, 76, 71, 71, 68, 65, 63, 62, 55, 53, 52, 49, 43, 40, 16

(2) Statistics:
(a) maximum = 97, (b) minimum = 16, (c) average(sample mean) = 68.7

(3) More statistical informations [2]:

- (a) *Dot diagram*
- (b) *Histogram* [3]
- (c) *Stem and leaf display*

Figure 1.2: Dot diagram for scores achieved on an examination

Figure 1.3: Histogram of examination scores

[2] Informations such as whether the graded points are evenly distributed about the sample mean or not, and whether there exists a extreme-value score among the graded points; we call the "outlier" and usually exclude it before computing such quantities as the sample mean (for example, in figure skating competitions)

[3] There occurs loss of informations on individual scores, in contrast to the dot diagram.

Decade	Unit					
0 - 9						
10 - 19	6					
20 - 29						
30 - 39						
40 - 49	0	3	9			
50 - 59	2	3	5			
60 - 69	2	3	5	8		
70 - 79	1	1	6	6	7	
80 - 89	0	0	3	4	5	7
90 - 99	2	3	7			

Table1.1 Stem-and-leaf display of 25 examination scores

1.3 Simulation of Random Phenomena

Definition 1.3 Pseudorandom number ganarator*:*
A computer program that generates sequences of numbers which appear to be random, but are really generated by a deterministic algorithm

1. Useful and essential for various computer simulations
2. Often comes with high level languages $\ni$: C, FORTRAN, and mathematics packages $\ni$: MATLAB, Mathcad

Example 1.3

MATLAB: generates 1000 random numbers $X \sim U(0,1)$

$$X = rand(1, 1000);$$

Example 1.4

Mathcad: generates 1000 random numbers $X \sim U(0,1)$

$$i = 0...999 \quad X_i = rnd(1)$$

Properties of GOOD pseudorandom number generator:

1. Different seeds produce different sequence of numbers
2. The numbers are *uniformly* distributed b/w 0 and 1 (or other specified interval)
3. Any specific number x_i is *uncorrelated* with other numbers in the chain

Creation of arbitrary distribution interval:

Given that $X \sim U(0,1)$, then

$$Y_i = a + (b-a)X_i$$

generates another sequence of random numbers $Y \ni: Y \sim U(a,b)$

Example 1.5

Suppose $X \sim U(0,1)$, then by way of the following conversion, a new sequence of random numbers W will be uniformly distributed b/w 4.9 and 5.1

$$W = 4.9 + 0.2X$$

(cf) Refer Table1.2 for sample MATLAB source code, Table1.3 for the resulting sequence of random numbers and corresponding statistics, and Figure1.4 for the histogram