Laboratory Experiment #4

Computing Auto and Cross Correlograms in the Frequency Domain

PURPOSE:

Implement a procedure for calculating the auto and cross correlograms using the Fourier coefficients of the signals to be correlated.

DESCRIPTION:

The cross correlation function for periodic signals $x(t)$ and $y(t)$ is defined in the time domain as:

$$R_{xy}(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)y(t+\tau)dt \tag{0.1}$$

where T is the least common period between $x(t)$ and $y(t)$. The resulting cross correlation function $R_{xy}(\tau)$ is also periodic (T), and the Fourier coefficients $C_{R_{xy}}(k)$ of $R_{xy}(\tau)$ can be formed from:

$$C_{R_{xy}}(k) = C_x^*(k)C_y(k)$$

where $C_x(k)$ and $C_y(k)$ are the Fourier series coefficients of the signals $x(t)$ and $y(t)$ respectively. This is the cross power spectrum for signals $x(t)$ and $y(t)$.
In a similar manner, the autocorrelation function of a periodic signal $x(t)$ with the fundamental period of T is as follows:

$$R_{xx}(\tau) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)x(t+\tau)dt \tag{0.2}$$

The Fourier coefficients $C_{R_{xx}}(k)$ of $R_{xx}(\tau)$ is then:

$$C_{R_{xx}}(k) = C_x^*(k)C_x(k) = |C_x(k)|^2$$

PROCEDURE:

1. Show that the Fourier coefficients of the cross correlation function can be expressed in terms of the Fourier coefficients of the signals to be correlated.

2. For $x(t)$, which is a three cycle sine wave, and $y(t)$, which is a one period even function cosine square wave, form the complex product:

$$C_R(k) = C_x^*(k)C_y(k)$$

3. Use the 5 largest magnitude coefficients and "WAVSYN" to construct an approximation of

   (a) the cross correlogram using $C_R(k)$.

   (b) the auto correlogram of $x(t)$ using $C_x^*(k)C_x(k)$.

   (c) the auto correlogram of $y(t)$ using $C_y^*(k)C_y(k)$.

4. Use equation (0.1) to verify these results by hand calculations of the cross correlogram and both auto correlograms. Compare a few points obtained in steps #3 and #4.