Contents

Chapter 1

INTRODUCTION

1.1 Overview

Signals and Systems

The *theory*, *analysis*, and *design* of signals and systems. [1]
$\Longrightarrow$ applied to many fields of engineering, science, economics and so on...

Objective:
Design signals and systems in order to send *fast* and *accurately* the desired information to the destination.[2]

Example 1.1

(Systems)	*(Signals)*
Radio and TV	music, voice, and image
Telephone networks	voice, speech, data
Radar and sonar	microwave, laser, sonic wave (target location)
Biomedical instrumentation	ECG, CT/MRI images (diagnosis)
Remote sensing	microwave, laser (target identification)
Seismic analysis	seismic signal (epicenter location)
Microphone	acoustic pressure $\rightarrow$ electrical signal
Loudspeaker	electrical signal $\rightarrow$ acoustic pressure
Sales analysis	previous sales $\rightarrow$ prediction (product control)
Weather forecast	previous data(temp.,rainfalls etc) $\rightarrow$ forecast

† RADAR: RAdio Detection And Ranging
‡ SONAR: SOund NAvigation Ranging

[1] In a wider sense, signals are the carriers of *information*, whereas the systems are the pathways of *information*. In addition, information itself can be regarded as signal.
[2] We also need *signal processing* as well.

Note: Human can also be considered as a system:

Example 1.2

(sensors)	*(corresponding signals)*	*(response)*
eye	scene	?
ear	music, speech	?
nose	smell	?
tongue	taste	?
hand	touch(feel)	?

where response depends on the signal processing of each individual's CPU, i.e. the brain!!!.

Example 1.3

In baseball game, players and manager exchange signals.

(a) encription

(b) simplification

(c) accuracy

Definition 1.1 SIGNALS :
Input, *output*, and *internal functions* that systems process and produce $\ni$: voltage, current, pressure, brightness(intensity), and displacement etc.

(cf.) Mostly, the independent variable for the signal functions is *time*, (but not necessarily!), for example two-dimensional images are functions of x and y coordinates. If the image is a moving image(e.g. video), t must be added.

Definition 1.2 SYSTEMS :
Devices, *processes*, and *algorithms* that given an input signal $x(t)$, produce an output signal $y(t)$.

Block Diagram(of signals and systems)

$$y(t) = T[x(t)]$$

Figure 1.1: Representaion of signals and systems #1

(cf) Multiple input/output system

$$\vec{y}(t) = T[\vec{x}(t)]$$

where

$$\vec{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$$
$$\vec{y}(t) = [y_1(t), y_2(t), \cdots, y_m(t)]^T$$

Figure 1.2: Representaion of signals and systems #2

Note:
Signals and systems are very closely inter-related to each other !!! And so are their definitions.

Three typical prblems(related to signals and systems)

(1) Know $x(t)$ and $T[\cdot]$, what is $y(t)$?	Analysis	(RLC circuit)
(2) Know $x(t)$ and $y(t)$, what is $T[\cdot]$?	Identification	(black box)
(3) Know $y(t)$ and $T[\cdot]$, what is $x(t)$?	Synthesis	(*digital* communication)

(cf) Synthesis: Instrumentation or recognition

Systematic way of solving engineering problems

Figure 1.3: Flow diagram of engineering problem solving

Mathematical Model(to make above problems tractable):
Mathematical equations representing *signals* and *systems* to provide *quantitative* analysis on *signal characteristics* and *system performance*

Example 1.4

Household voltage: Signal

$$v(t) = 220\sqrt{2}\cos[2\pi 60t] \quad -\infty < t < \infty$$

where $220\sqrt{2}$ is the peak-to-peak voltage whereas the r.m.s.(root mean square) voltage is given by $V = \sqrt{\frac{1}{T}\int_T v^2(t)dt} = \sqrt{\frac{220^2 \cdot 2}{2}} = 220$.

Example 1.5

Resistor: System

$$v(t) = R \cdot i(t)$$

Note: The mathematical models of above examples are NOT exact, since

(1) Due to interference noises, $v(t)$ is not perfectly sinusoidal

(2) The resistance R is not constant, and it depends on ambient temperature

Therefore, more accurate mathematical models for the above two examples should be:

1. $v(t) = \{220\sqrt{2} + \Delta v(t)\} \cos[2\pi(60 + \Delta f(t))t]$
2. $v(t, T) = R(T) \cdot i(t)$, where T is the ambient temperatue.

$$\Downarrow$$

Practically impossible to describe any physical phenomenon into a perfect mathematical model.

OBJECTIVE:
Make the mathematical models as *simple* **as possible, but still** *retain those parameters* **that significantly affect the characteristics of signals and systems.**

(e.g.) above examples

Typical categorization of signals and systems (continuous vs. discrete)

Definition 1.3 A CONTINUOUS-time signal has a value defined for each point in time, and a continuous-time system *operates on* and *produces* continuous-time signals

Example 1.6

continuous-time signal

Figure 1.4: Typical continuous-time signal $x(t)$.

Definition 1.4 A DISCRETE-time signal has a value defined only at discrete points in time, and a discrete-time system *operates on* and *produces* discrete-time signals

Example 1.7

discrete-time signal

Figure 1.5: Typical discrete-time signal $x[n]$.

Note:

(1) Discrete-time signal is a *sequence of values*

(2) Discrete-time signals are usually obtained by *sampling* continuous-time signals

(3) Sample period needs not be constant, but in most of the cases, we consider discrete signals equally spaced in time!!! (*uniform sampling*), i.e., $x_T(t) = x[n]$ where T is constant

Definition 1.5 A DIGITAL signal is a discrete-time signal which may take on only a set of countable values, i.e., it is a discrete-time *quantized* signal, and a digital system *operates on* and *produces* digital signals

Example 1.8

discrete-time signal $x_d[n]$ takes on only one of {-2, -1, 0, 1, 2 }

Figure 1.6: Typical digital signal $x_d[n]$.

Note:
If you want to represent a discrete-time signal $x[n]$ in a 8-bit computer machine, we only can use $2^8 = 256$ levels for the value of $x[n]$.
$\Rightarrow$ *Quantization error* occurs!!!

Example 1.9

Conversion process from continuous-time signal to digital signal via discrete-time signal

Figure 1.7: Conversion of signals: $x(t) \rightarrow x[n] \rightarrow x_d[n]$

1.2 General Characteristics of Signals

Category of Signals(in terms of their characteristics)[3]

1.2.1 Deterministic versus Random Signals

Definition 1.6 A DETERMINISTIC signal behaves in a fixed *known* way w.r.t. [4] time, thus can be modeled as a known function of time t.

(cf.) For a fixed time t, $x(t)$ is completely known.

Definition 1.7 A RANDOM signal takes on one of many possible values at each time (for which the signal value is defined), and it requires a *probabilistic model* to be described.

Example 1.10

$x(t) = \sin(t)$: Deterministic

$n(t)$ from random signal generator: Random

[3]In addition to continuous versus discrete discussed in previous section
[4]w.r.t.: with respect to

1.2.2 Periodic versus Aperiodic(Non-periodic) Signals

Definition 1.8 A continuous-time signal $x(t)$ is periodic if and only if (iff):

$$x(t+T) = x(t) \quad \forall t$$

where T is the period of the signal $x(t)$

(cf.) Likewise, the periodic discrete-time signal is defined as follows:

Definition 1.9 A discrete-time signal $x[n]$ is periodic if and only if (iff):

$$x[n+N] = x[n] \quad \forall n$$

where N(integer) is the period of the signal $x[n]$

Note:
The smallest value of T and/or N for which above definitions hold is called the *fundamental period* and usually denoted as T_0 and N_0.

Example 1.11

Fundamental period of continuous sinusoid

Figure 1.8: Illustration of the fundamental period of continuous sinusoidal signal $x(t)$.

1.3 General Characteristics of Systems

Category of Systems(according to their characteristics)

1.3.1 Static versus Dynamic Systems

Definition 1.10 A STATIC system is a system with an output signal which, at any specific time, depends on the value of the input signal *at only that time*

$$y(t_0) = f[x(t_0)]$$

Figure 1.9: Static system

Note:
In a static system;

(1) There ∃ NO energy starage elements or memory such as inductor, capacitor etc.

(2) There ∃ NO integrals, derivatives, or signal delay

Definition 1.11 A DYNAMIC system is a system with an output signal which, at any specific time, depends on the value of the input signal *at both the specific time and at other times*

$$y(t_0) = f[x(t_0), x(t_1), x(t_2), \ldots]$$

Figure 1.10: Dynamic system

Definition 1.12 A CAUSAL system is a system for which the output signal at a specific time depends only on the input signal *at times preceding or equal to the specified time*

$$y(t_0) = f[x(t_0), x(t_1), x(t_2), \ldots], \quad \text{where } t_i < t_0 \;\; \forall i = 1, 2, \ldots$$

Figure 1.11: Causal system

Note:
A static system is always a causal system!!!

Example 1.12

$$y(t) = \int_{-\infty}^{t} x(\alpha) d\alpha \qquad \text{: causal}$$

Example 1.13

$$y[n] = \alpha x[n] + \beta x[n+1] \qquad \text{: non-causal}$$

Figure 1.12: Relation among systems

1.3.2 Linear Systems

Definition 1.13 A system is called a LINEAR system if it satisfies the following condition:

$$T\left[\sum_{i=1}^{N} a_i x_i(t)\right] = \sum_{i=1}^{N} a_i T\left[x_i(t)\right] \triangleq \sum_{i=1}^{N} a_i y_i(t) \tag{1.1}$$

where a_i's are constants.

$$y(t) = T[x(t)]$$

Figure 1.13: Linear system

1.3.3 Time Invariant Systems

Definition 1.14 A system is called a TIME-INVARIANT system if it satisfies the following condition:

$$T\left[x(t-t_0)\right] = y(t-t_0) \tag{1.2}$$

Example 1.14

Determine whether each of the follwing system is *linear* and/or *time invariant*:

(1) $T[x(t)] = x^2(t)$
(2) $T[x(t)] = ax(t) + b$
(3) $T[x(t)] = \sin\{x(t)\}$
(4) $T[x(t)] = e^{-x(t)}$
(5) $T[x(t)] = ax(t-t_0)$
(6) $T[x(t)] = x(t^2)$
(7) $T[x(t)] = e^{-t}x(t)$

1.3.4 Linear Time Invariant(LTI) Systems

Definition 1.15 A system is called an LTI system if it is both *linear* and *time-invariant*, i.e. if both (1.1) and(1.2) are satisfied simultaneously!!!

(cf) The definition of LTI system for discrete-time case is similar, and will be discussed in the later part of this course.

Course Schedule:
We will first discuss the continuous-time signals and systems, and then move on to the discrete-time signals and systems...